The method for determining angular stability factors based on power waveforms

Paszek Stefan, Nocoń Adrian

Abstract
The paper presents the method for numerical determination of electromechanical eigenvalues of the system state matrix on the basis of the recorded instantaneous power waveforms in generating units or transmission lines. The sequence, hybrid algorithm being the combination of the genetic algorithm and the Newton gradient algorithm with limitations was used in computations. The electromechanical eigenvalues computed are the basis for determining the PS angular stability factors introduced.

Keywords: power system stability, instantaneous power waveforms, stability factors, distributed sources

Introduction

Disturbances occurring during the exploitation of a power system (PS) result in appearing slow-variable, oscillatory changes of the synchronous generator rotational speed, i.e. electromechanical swings. These swings also appear in instantaneous power waveforms (called conventionally active power waveforms) in different system sites, among others generating units and transmission lines. Their characteristic feature is low frequency (from the range of 0.1÷2 Hz) and relatively small damping decrement. The possibility of the appearance of weakly damped electromechanical swings increases in large power systems. It is the consequence of the increase in the number of:

- synchronous generators of high rated power characterised by large relative synchronous reactance,
- static excitation sources controlled by means of fast voltage regulators of large gain,
- high voltage transmission lines of significant length,
- distributed sources.

These swings limit the possibility of electrical energy transmission and, under unfavourable conditions, they can result in the loss of the system stability. That is why it is important from both theoretical and practical point of view to measure and analyse power swings occurring in PS.

The determination of the stability factors is more and more significant because of the continuous increase in the number and unitary power of distributed sources (especially wind farms) installed in the system. The transient states caused by unstable operation of the distributed sources, in particular by switching them off, can result in significant power swings of the generators installed near these sources. So the determination of the stability is necessary for correct planning the investments connected with the distributed power engineering.

1. Stability factors

When analysing the instantaneous power transient waveforms in generating units and transmission lines, it is possible to separate in them the modal components connected with particular eigenvalues of the state matrix of the PS linearised about the steady operating point. The modal components related to the electromechanical complex eigenvalues $\lambda_h = \alpha_h \pm \mathrm{j}\,\nu_h$ (Paszek 1998) are of crucial significance. They can either decay with or grow, dependently on the eigenvalue real part. Even if there is $\mathrm{Re}\{\lambda_h\} > 0$ for only one eigenvalue, the system becomes unstable.

It is possible to estimate approximately that the satisfactory decay of electromechanical swings in the system is obtained when the real parts of all eigenvalues are lower than, for instance, −0.3 (it is a conventional, estimated value). The setting time of the waveforms expressed by the inequality $t_{ust} < 13$ s corresponds to this value. Also other criterion conditions that should be met by the modal waveforms in PS can be worked out taking into account the relative or logarithmic damping coefficient connected with the electromechanical eigenvalues. Owing to the above, the following power system angular stability factors are proposed:

$$W_1 = \max(\alpha_h) \tag{1}$$

$$W_2 = \max(\xi_h) = \max\left(\frac{\alpha_h}{\sqrt{\alpha_h^2 + \nu_h^2}}\right) \tag{2}$$

$$W_3 = \min(\eta_h) = \min\left(\ln\left(2\pi\frac{-\alpha_h}{\nu_h}\right)\right) \tag{3}$$

2. Determination of electromechanical eigenvalues

In order to determine the values of the PS angular stability factors proposed above, it is necessary to calculate the electromechanical eigenvalues. They can be computed on the basis of the state matrix of the system linearised about the operating point. They can also be computed by means of specialized algorithms - for instance AESOPS, PEALS (Sauer et al 1991) - basing only on the state equations of the particular generating units and the algebraic equation of the power network without the necessity of determining the whole state matrix of the system. However, in both mentioned cases there are great difficulties in determining the reliable mathematical models of the system elements and in working out the set of the real parameters of these models.

That is why it seems useful to develop a method for computing the electromechanical eigenvalues based on measurements of the real waveforms appearing in generating units and transmission lines after the occurrence of different disturbances (introduced on purpose or random) of the system steady state. The new possibilities of taking these measurements when using the technology WAMS (wide area measuring systems) make it even more purposeful.

In the paper it is assumed that an impulse change of the reference voltage in the excitation system of a selected synchronous generator $\Delta U_z(t) = \Delta U_z \delta(t)$ is introduced to the multimachine PS. There are analysed the waveforms of the generator instantaneous power deviations (from the steady state) in the particular generating nodes of the system. At the single eigenvalues of the state matrix, these waveforms (for the system model linearised about the operating point) can be presented in the form (Kudła and Paszek 1995):

$$\Delta \boldsymbol{P}(t) = \sum_{h=1}^{N} \boldsymbol{C}\boldsymbol{V}_h e^{\lambda_h t} \boldsymbol{W}_h^T \boldsymbol{B} \Delta \boldsymbol{U}_z \qquad (4)$$

where: $\Delta \boldsymbol{P}$, $\Delta \boldsymbol{U}_z$ - vectors of deviations of the instantaneous power and reference voltage (impulse change) in particular generating units, $\boldsymbol{V}_h$, $\boldsymbol{W}_h$ - right- and left-hand eigenvectors of the state matrix corresponding to the *h*-th eigenvalue, $\boldsymbol{C}$, $\boldsymbol{B}$ - matrices of the system output and input.

From the theoretical point of view, all modal components connected with the particular eigenvalues of the state matrix occur in the waveform of each state variable and each output quantity. In practice, when analysing the instantaneous power waveforms in the particular generating units within the time range, for instance (1-10 s), after appearing the disturbance, only several (usually from 1 to 5) significant modal components connected with the electromechanical eigenvalues can be observed. The amplitudes of other components are either very small or they decay quickly after the disturbance – the real parts of the appropriate eigenvalues are negative and their modulus is large. On the other hand, the modal components related to the particular electromechanical eigenvalues have significant amplitudes usually in two, three instantaneous power waveforms of the particular generating units. Bearing that in mind, by analysing and decomposing the instantaneous power waveforms in the particular generating units, it is possible to compute all electromechanical eigenvalues of the system. The number of the electromechanical eigenvalues (of the positive imaginary part) is equal to the number of the PS generating units minus one.

3. Determination of electromechanical eigenvalues

Exemplary computations were carried out for a 7-machine Cigre PS shown in Fig. 1.

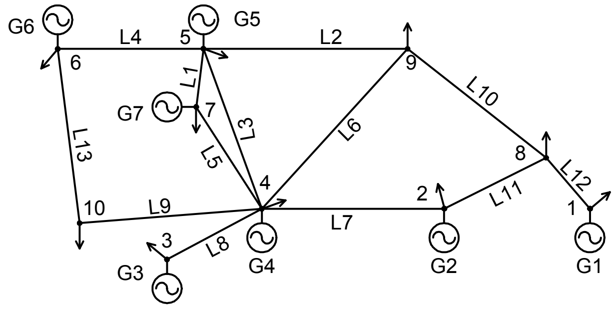

Fig.1 Cigre power system

A mathematical model of a power system in the Matlab–Simulink environment was constructed by connecting the generating units via the reduced power network. A general model of the generating unit is presented in Fig. 2. In this model, using the *"Configurable Subsystems"* type blocks, it is possible to create a specific model of the generating unit when choosing the specific model of: a synchronous generator, an excitation system, a turbine and PSS (Paszek and Pawłowski 2007).

It was assumed in computations that all synchronous generators were represented by a non-linear model of a GENROU turbogenerator (De Mello and P. Hannett 1986). Four equivalent electric circuits in the rotor, two in the *d* axis (excitation circuit and one damper circuit) and two in the *q* axis (two damper circuits) correspond to this model. It was assumed that the excitation systems were represented by a nonlinear model of the Polish national static excitation system, and the turbines by an IEEEG1 steam turbine model (Paszek and Pawłowski 2007). The system stabilizer PSS3B was introduced to the first generating unit.

The disturbance was assumed to be an impulse change of the reference voltage in the synchronous generator excitation system of the first generating unit $\Delta U_z(t) = \Delta U_z \delta(t)$, $\Delta U_z = 0.03 * U_{zu}$, U_{zu} - reference voltage in the steady state.

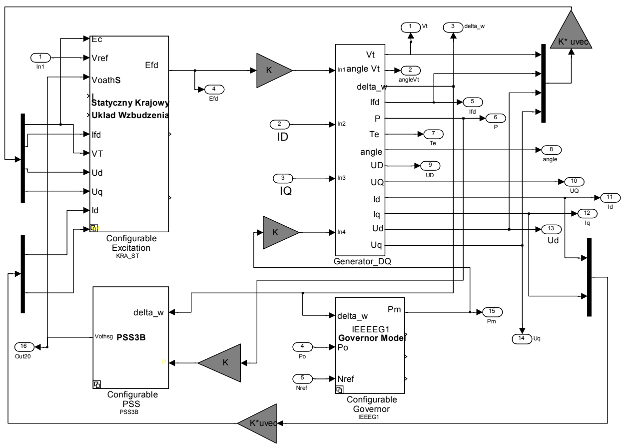

Fig.2 Structural model of generating unit in the Matlab–Simulink environment

Linearization of the state equations of the system analysed was performed in the Matlab-Simulink environment. There were computed: the state matrix (of dimension 95×95), matrices $\boldsymbol{B}$ and $\boldsymbol{C}$, the eigenvalues (including six electromechanical ones presented in Tab. 1), the right- and left-hand eigenvectors as well as the instantaneous power waveforms in the particular (*i*-th) generating units which can be approximately given by the following relationship:

$$\Delta P_i = \sum_{h=1}^{n} 2|A_{ih}| e^{\alpha_h t} \cos(\nu_h t + \arg(A_{ih})) \qquad (5)$$

Only the significant modal components related to the electromechanical eigenvalues (Tab. 1) are taken into account in Eq. (5), while the complex amplitudes of the waveforms A_{ih} result from Eq. (4) when taking into account the complex, conjugate electromechanical eigenvalues. Tab. 2 gives the values of the significant complex amplitudes of the instantaneous power waveforms in the particular generating units which are taken into account in the computations.

	Eigenvalue
λ_1	-0.8038 +j 8.6480
λ_2	-0.7159 +j 10.0585
λ_3	-0.5333 +j 10.4212
λ_4	-0.3795 +j 9.3543
λ_5	-0.2713 +j 6.4525
λ_6	-0.1507 +j 7.8709

Tab.1 Electromechanical eigenvalue

	Significant modal component
ΔP_1	$A_{11}= \|1.4741\| e^{-j0.0991}$, $A_{15}= \|3.5464\| e^{j\,2.3828}$,
ΔP_2	$A_{21}= \|1.7005\| e^{-j2.8272}$, $A_{24}= \|1.7628\| e^{-j0.5444}$, $A_{25}= \|4.0472\| e^{j\,1.8983}$,
ΔP_3	$A_{34}= \|2.0176\| e^{-j0.7583}$, $A_{35}= \|1.4189\| e^{-j0.9010}$, $A_{36}= \|5.9601\| e^{j\,2.5631}$,
ΔP_4	$A_{43}= \|3.9529\| e^{j3.0581}$, $A_{44}= \|3.6418\| e^{j2.2687}$, $A_{45}= \|1.4498\| e^{-j\,0.7974}$, $A_{46}= \|1.0925\| e^{j\,2.3524}$,
ΔP_5	$A_{52}= \|1.3329\| e^{j3.1271}$, $A_{53}= \|1.0031\| e^{-j0.0977}$, $A_{54}= \|10.2133\| e^{j\,2.7485}$, $A_{55}= \|1.6886\| e^{-j\,0.9303}$, $A_{56}= \|2.2702\| e^{-j\,0.4130}$,
ΔP_6	$A_{62}= \|1.0552\| e^{j0.0363}$, $A_{64}= \|3.6457\| e^{j\,2.8603}$,

Tab.2 Significant modal components

The electromechanical eigenvalues λ_h, as well as the values of the significant complex amplitudes A_{ih}, can be determined numerically by analysing the recorded instantaneous power waveforms in the particular generating units. The problem of determining λ_h and A_{ih} values was brought to the minimisation of the objective function describing the differences between the true (recorded at the investigated node of the network) and approximating waveform. In the paper the waveforms obtained from simulations on the basis of the equation (5) are assumed to be the true ones. The objective function being minimised is a function of several variables (searched parameters), the number of which depends on the number of the significant modal components present in the waveform. Each modal component depends on four real parameters (the real and imaginary part of the eigenvalue λ_h as well as the modulus and argument of the complex amplitude A_{ih}). In the Cigre system considered two significant modal components occur in the instantaneous power waveform of the first node, so this waveform is determined by eight real parameters, whereas there are five components in the power waveform of the fifth node, which means twenty real parameters (Tab. 2).

The reduction in the number of the searched variables (parameters) at the successive stages was achieved by taking into account in computations the eigenvalues determined previously. For instance, when analysing the waveforms at the second node it can be assumed that the eigenvalues λ_1 and λ_5 were previously determined correctly by analysing the waveforms at the first node of the PS investigated. Such a solution requires, however, an additional analysis of the system investigated, for instance by using the method of participation factors (Paszek 1998).

In the investigations presented the objective function was minimised by using the sequence, hybrid algorithm being the combination of the genetic algorithm (Rutkowska et al 1999) and the Newton gradient algorithm with limitations.

The genetic algorithm belongs to global minimisation algorithms (Rutkowska et al 1999) and does not require the determination of the starting point, only the search region. Its disadvantage is a not precisely determined stopping criterion. On the other hand, the Newton algorithm is used for searching the local minimum and the result depends significantly on the starting point assumed (Rade and Westergren 2004). If these two algorithms are combined in such a way that the genetic algorithm determines the starting point for the Newton algorithm, their basic disadvantages can be eliminated.

The exemplary instantaneous active power waveforms for the system investigated are shown in Fig. 3. There are marked the bands of the power waveforms limited by the waveforms of the maximum and minimum values of the power determined according to the relationship (5) for the maximal errors obtained at the first optimisation stage by means of the genetic algorithm.

In the computations there were assumed the following search regions for variables:

- for Re{λ_h} – (-0.9 ÷ 0.1 1/s),
- for Im{λ_h} – (0.63 ÷ 12.57 1/s), which corresponds to the swing frequency from 0.1 Hz to 2 Hz.

In the example presented there were carried out the repeated computations of the searched coefficients for particular PS generating units (200 trials for each generating node). On that basis the statistic analysis was performed and the maximal errors for the particular eigenvalues were determined. The results obtained are shown in Fig. 4. Since the waveforms got from the computer simulation were assumed to be the true ones in the computations, the errors obtained were small. That is why the value of the error logarithm for the particular modal values is presented in Fig. 4.

Since the modal components can appear in different network nodes, the error values obtained for the particular generating units are marked in Fig. 4.

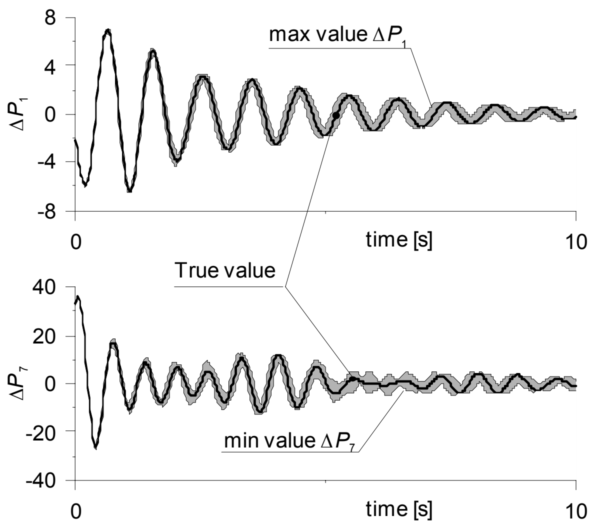

Fig.3 Instantaneous power deviations in first and seventh generating unit

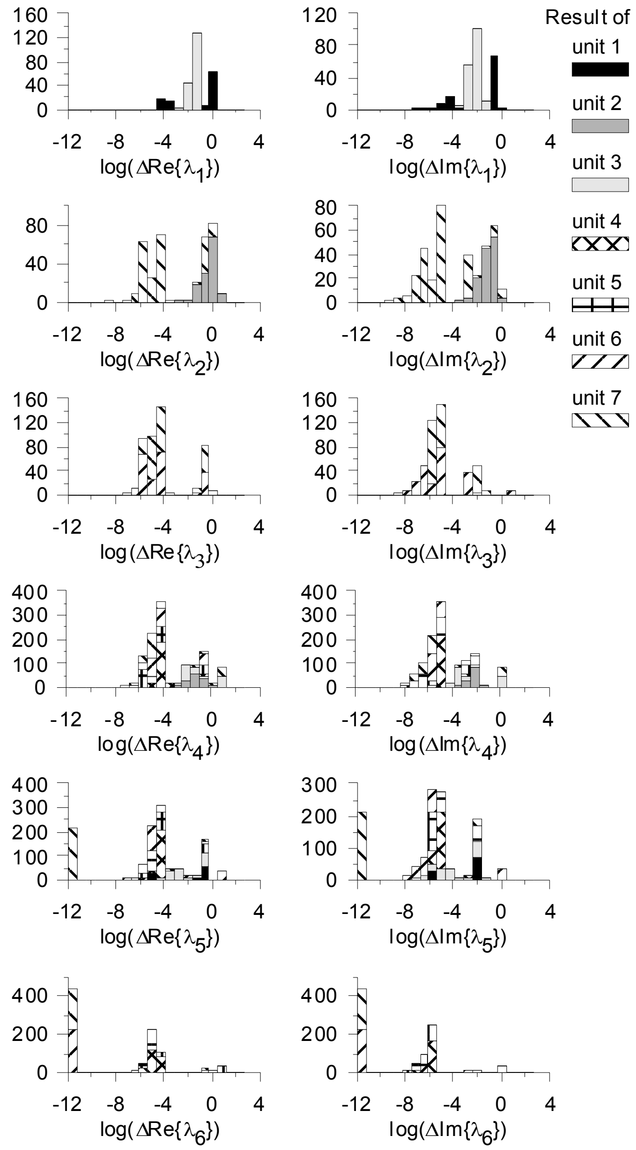

Fig.4 Histograms of error logarithm of determining the searched modal components

The values of the PS angular stability factors were computed on the basis of the modal values calculated numerically according to the relationships (1) ÷ (3). They are given in Tab. 3.

	The worst value computed	True value	Error
W_1	-0.150246	-0.150700	0.30%
W_2	-0.019119	-0.019143	0.12%
W_3	-2.118979	-2.117759	-0.05%

Tab.3 Angular stability factors

Conclusion

On the basis of the computations presented, it can be stated that:

- the error values obtained for the particular electromechanical eigenvalues do not exceed 2% (Fig. 4),
- the errors of determining the eigenvalues result in the errors of the angular stability factor values not larger than 0.5% for the factor W_1 and 0.2% for the other factors.

Summing up, the proposed method for numerical determination of electromechanical eigenvalues and modal components based on the approximation of instantaneous power waveforms gives the correct results.

The proposed method for determination of the stability factors can be used for the analysis of the stability of the system in which there is installed the significant number of distributed sources.

Acknowledgement

The research is realised in the framework of the project „Power Security of Poland“ (PBZ-MEiN-1/2/2006) carried out by the Consortium of Universities of Technology of Gdańsk, Gliwice, Warszawa and Wrocław.

References

[1] DE MELLO, F., HANNETT, P. L. H. (1986): Representation of Saturation in Synchronous Machines. IEEE Transactions on Power Systems, Vol. PWRS-1, No. 4, pp. 8-18.

[2] KUDŁA, J., PASZEK, S. (1995): Reduced-order transfer function in power system (in Polish). 18-th International Conference on Fundamentals of Electrotechnics and Circuit Theory, SPETO'95, Vol. 2, pp. 299-304.

[3] PASZEK, S. (1998): Optimisation of the power system stabilizers in a power system (in Polish). Proceedings of the Silesian Technical University ELEKTRYKA No. 161, Gliwice, Poland.

[4] PASZEK, S., PAWŁOWSKI, A. (2007): Parameter optimisation of power system stabilizers PSS2A in non-linear multi-machine power system. 30-th International Conference on Fundamentals of Electrotechnics and Circuit Theory, SPETO'2007, pp. 169-170.

[5] RADE, L., WESTERGREN, B. (2004): Mathematics Handbook for Science and Engineering, Springer-Verlag.

[6] RUTKOWSKA, D., PILIŃSKI, M., RUTKOWSKI, L. (1999): Neural networks, genetic algorithms and fuzzy logic systems (in Polish), Wydawnictwo Naukowe PWN.

[7] SAUER, P. W., RAJAGOPALAN, C., PAI, M. A. (1991): An explanation and generalization of the AESOPS and PEALS algorithms. IEEE Transactions on Power Systems, Vol. 6, No. 1, pp. 293-299.

Paszek Stefan
Nocoń Adrian

Silesian University of Technology
Faculty of Electrical Engineering
Akademicka 10, 44-100 Gliwice
Tel.: (+48) 32 237-14-47
E-mail: Stefan.paszek@polsl.pl, Adrian.nocon@polsl.pl