Function approximation and digital linearization in sensor systems

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Abstract
The aim of this article is to expose the developed method for solution of experimentally obtained sensor characteristic approximation in microcomputer. This process should fulfill demands given by claims on data processing, for example linearization of sensor characteristic. Errors in the system were analyzed – error of method, rounding error, influence of ADC and the appropriate linearization method choosing technique was designed. The comparison of efficiency of methods was executed in several examples.

Key words: smart sensor systems, sensor signal processing, sensor characteristic approximation, sensor characteristic linearization

Introduction

With higher quality of each technical element in the control system structure higher demands are put on process value measurements. Today measurement devices should be able to work with correspondent metrological and functional qualities. This is achieved by using microcomputer in the measurement channel structure, fig.1.

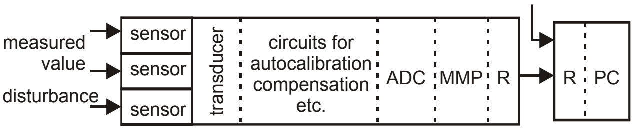

Fig.1 Intelligent measure element

Smart sensor system (SSS) is autonomy digital system characterized by primary data processing (PSI), diagnostic and auto-calibration functions and communication control. The advantages of SSS are especially higher quality (better evaluation of original signals obtained from sensing element, better functional qualities, higher safety, less demands on communication system) and then more effective performance of digital (local or total) control centre. Because data from intelligent measurement element are primary processed, they give actual, accurate and reliable (clean) information about the state of controlled process (process value) and this accelerates realization of control algorithm and reaction to physical values.

Besides acquiring of output signal from sensor element and AD conversion, there are implemented these functions of PSI into SSS in particular: linearization of sensor characteristics, filtration of incoming signal, reduction of measured values, disturbance correction, dynamic error correction, indirect measurement, other computations etc.. In these cases there are implemented mathematical functions in the microcomputer. Therefore accurate and demands meeting realization of these functions is important. Considering limitations of digital processing in SSS, e.g. small memory, small computing capacity, it is inevitable to use approximations in many cases. Approximation is especially necessary, if the real mathematical description of realized function is unknown. Using approximation for digital linearization of sensor characteristics will bee discussed.

1. Linearization of sensor characteristics

Natural feature of many sensors is their nonlinear characteristic. In SSS digital linearization is fundamentally used, but SSS producers usually reduce the nonlinearity in analog part of measurement channel too. More important is to ensure repetition of the analog part. In the fig.2 there is outlined process of linearization using the inverse sensor characteristic.

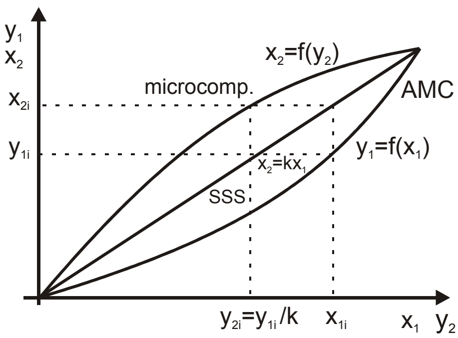

Fig.2 Linearization of sensor characteristic

Sensor characteristic is $y_1 = f(x_1)$ in the microcomputer it is embedded inverse characteristic $x_2 = f(y_2)$ so that it makes the whole characteristic to be linear $x_2 = kx_1$.

For the linearization in microcomputer the approximation errors by different approximation methods – error of method – and also rounding error and influence of AD converter should be analyzed.

2. Methods for mathematical function approximation in the microcomputer memory

Consider n+1 number of values (arguments) $x_0 < x_1 < x_2 <...< x_n$ and functional values $y_0 = F(x_0)$, $y_1 = F(x_1)$, $y_2 = F(x_2)$, ... , $y_n = F(x_n)$, where F is unknown real function. The task is approximation of the function F. Several methods can be used for this purpose. The method selection depends on our knowledge about the function besides the function values in given arguments. Two approaches are distinguished. In this paper Newton interpolation polynomials (NIP) are used with respected nodes.

2.1 Respecting of nodes

In this case approximation consistently respects the given values $y_0, y_1, y_2,...,y_n$ in arguments $x_0, x_1, x_2,...,x_n$, named also nodes or poles (fig.3). This is actually interpolation, which is approximation between nodes. If the approximation function *Fa* is used outside of the interval $\langle x_0, x_n \rangle$, it is called extrapolation. Interpolation (extrapolation) is used, if errors in values y_i (acquired through experiment - measurement) could be neglected. For example values y_i are obtained by means of statistics methods (mean value), which is possible in easy and reproducible measurements such as pressure or position measurements. Next example is particularly accurate measurement. In this case it is also recommended to repeat the measurement more times. Error is limited with distance of nodes or information about the smoothness (shape) of the function. By sensor characteristic approximation for every interval $\langle x_0, x_1 \rangle$ (or for every two or three intervals depending on polynomial degree) other approximation function (polynomial) is usually employed. The whole approximation function is composed from these partial functions.

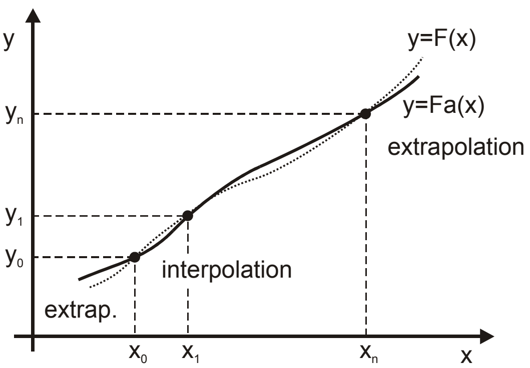

Fig.3 Approximation with respected nodes
***F* – unknown function, *Fa* – approximation function**

2.2 Unrespected nodes

This is the approximation, where function values y_i obtained by measurement are inaccurate or they are not respected because of other reasons and approximation function could go only near to them as depicted in the fig.4. The reason of inaccuracy can be a very difficult experiment or too low number of measurements. By this approximation method only one approximation function can be employed for the whole scale or for two or three intervals. There are several conditions used to decide about the best proximity of two functions. The best known condition is the least squares estimation.

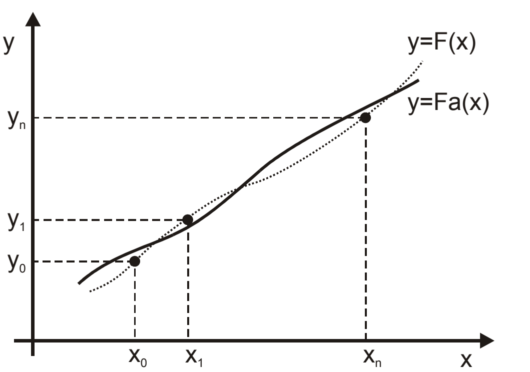

Fig.4 Approximation with not respected nodes
F – unknown function, Fa – approximation function

3. Newton interpolation polynomial

Interpolation polynomials are based on Lagrange polynomials. If it is supposed that difference between poles x_i is constant, it means the step

$$h = x_{i+1} - x_i \tag{1}$$

is constant, the notation of the polynomial can be adjusted into the form of Newton interpolation polynomial (NIP) $N_n(x)$ defined as

$$N_n(x) = y_0 + \frac{\Delta y_0}{1!h}(x - x_0) + \frac{\Delta^2 y_0}{2!h^2}(x - x_0)(x - x_1) + \dots + \frac{\Delta^n y_0}{n!h^n}(x - x_0)(x - x_1)\dots(x - x_{n-1}) \tag{2}$$

where $\Delta^k y_i$ is difference of degree k in the node x_i:

$$\Delta^k y_i = \Delta^{k-1} y_{i+1} - \Delta^{k-1} y_i = \sum_{j=0}^{k} \binom{k}{j} (-1)^j y_{i+1-j} \tag{3}$$

Disadvantage of polynomial interpolation is parasitic oscillation tendency when using higher number of nodes (higher degree of polynomial). This is the reason why polynomials of high degree are not used in sensor technique. For approximation of inverse sensor characteristic it is suitable only NIP of degree 0, 1 or 2, fig.5. Approximation function arises as a combination of required *l* polynomials of degree *m* for *n*+1 nodes

$$Fa(x) = N_{m,i}(x) = Fa_i(x); \quad x \in \langle x_i, x_{i+m} \rangle \qquad i = 0,1,\dots,l-1 \tag{4}$$

If *m*>0, number of polynomial *l* is

$$l = \frac{n}{m} \tag{5}$$

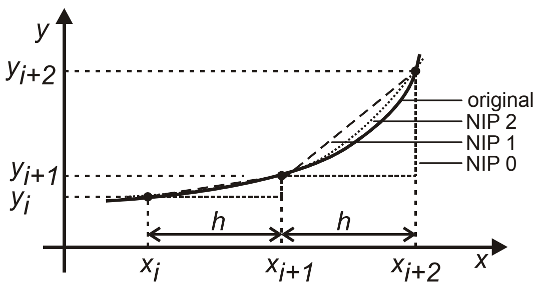

Fig.5 Approximations with NIP

3.1 NIP of degree 0

Newton interpolation polynomial of degree 0 (NIP 0, fig.5) means only directly measured values – poles

$$Fa_i(x) = N_{0,i}(x) = F(x_i) \tag{6}$$

This method is also called table method, where every remembered value $Fa_i(x) = F(x_i) = y_i$ is NIP of degree 0. Every value will be used for approximation on one interval therefore small adaptation must be made in the equation (4) (originally approximation function $Fa_i(x)$ is valid for $x \in \langle x_i, x_{i+0} \rangle$):

$$\begin{aligned} &Fa(x) = N_{0,i}(x) = Fa_i(x); \quad x \in \langle x_i, x_{i+1} \rangle \quad for \\ &i = 0,1,\dots,l-2 \\ &Fa(x) = N_{0,i}(x) = Fa_i(x); \quad x = x_n \quad for \quad i = l-1 \end{aligned} \tag{7}$$

Number of polynomials is $l=n+1$.

3.2 NIP of degree 1

Newton interpolation polynomial of degree 1 is linear equation given by two points

$$Fa_i(x) = N_{1,i}(x) = F(x_i) + (x - x_i)\frac{\Delta F(x_i)}{x_{i+1} - x_i}$$
$$\Delta F(x_i) = F(x_{i+1}) - F(x_i) \qquad (8)$$

It is interpolation with linear functions (fig.5)

$$Fa(x) = N_{1,i}(x) = Fa_i(x); \quad x \in \langle x_i, x_{i+1} \rangle$$
$$i = 0,1,\ldots,l-1 \qquad (9)$$

For this case number of polynomials is $l=n$.

3.3 NIP of degree 2

Newton interpolation polynomial of degree 2 is square function solved from three points (fig.5) and it holds:

$$Fa_i(x) = N_{2,i}(x) = F(x_i) + (x - x_i)\frac{\Delta F(x_i)}{x_{i+1} - x_i} +$$
$$+ (x - x_i)(x - x_{i+1})\frac{\Delta^2 F(x_i)}{2(x_{i+1} - x_i)^2} \qquad (10)$$
$$\Delta^2 F(x_i) = \Delta F(x_{i+1}) - \Delta F(x_i)$$
$$\Delta F(x_i) = F(x_{i+1}) - F(x_i)$$

Final approximation function is composed from l of these polynomials, while $l=n/2$:

$$Fa(x) = N_{2,i}(x) = Fa_i(x); \quad x \in \langle x_i, x_{i+2} \rangle$$
$$i = 0,1,\ldots,l-1 \qquad (11)$$

4. Errors of approximation in microcomputer

The basis for approximation process is required accuracy of approximation in microcomputer. Accurate class of digital processing Ac_C results from intended accurate class of whole digital measuring element (DME) Ac_{DME} and of analog measuring element (AME) Ac_{AME}, because for block scheme in the fig.6 it can be written (equation for serial element placement)

$$Ac_{DME} = \sqrt{Ac^2_{AME} + Ac^2_C} \qquad (12)$$

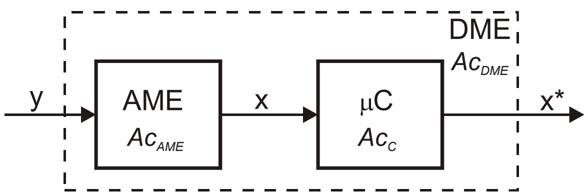

Fig.6 Block scheme of digital measuring element

while accurate class of approximation is

$$Ac_C = \frac{\Delta_C}{x^*_{FS}} 100\,[\%] \qquad (13)$$

where Δ_C is maximal approximation error and x^*_{FS} is full scale of microcomputer output value x* (because by linearization in the microcomputer maximal nonlinearity error of AME is reduced, this error must not be involved in Ac_{AME}). Maximal absolute error of digital processing Δ_C will be the input requirement.

On the other hand mainly approximation error Δ_{APP} should be taken into account, which is given by maximal deviation between approximation and approximated function. For Newton interpolation polynomials approximation error could be solved from the general relation for error of polynomial interpolation (polynomial of degree n) - this equation is derived from statement of Rolle from mathematical analysis - [3]

$$|E(x)| \le |(x - x_i)(x - x_{i+1})\ldots(x - x_{i+m})| \frac{M_{m+1}}{(m+1)!} \qquad (14)$$

where M_{m+1} could be determined as a maximal value of approximation function derivation $|F^{m+1}(\xi)|$ for the whole scale $\xi \in (x_0, x_n)$. In the sensor technique inverse sensor characteristic is usually unknown therefore the analytical formulation of derivation F^{m+1} of degree $m+1$ is unknown too. The value M_{m+1} could be estimated by means of difference $\Delta^{m+1}F(x_i)$ [3]. Maximal value E corresponds to the maximal error of mathematical approximation Δ_{APP}:

$$E_{max} = \max(|E(x)|) \qquad (15)$$

$$\Delta_{APR} = \max(|Fa(x) - F(x)|) \qquad (16)$$

and requirement is:

$$E_{max} \le \Delta_{APR} \qquad (17)$$

According to this the maximal approximation error Δ_{APP} could be estimated and expression between this error and the step h could be found. If the accuracy demands are high, it is better to use simulation.

Next error which should be taken into account is rounding error Δ_R. Influence of this error on the overall error Δ_C has to be investigated. As a limitation for overall interpolation or extrapolation error by approximation of function F with Lagrange polynomial it could be declared value [3]

$$R = H\Delta_R + |E| \qquad (18)$$

The condition should be fulfilled

$$R \le \Delta_C \qquad (19)$$

If the maximal rounding error is different from zero, for approximation error can be written

$$\max(|E|) = E_{max} \le \Delta_{APR} = \Delta_C - H\Delta_R \qquad (20)$$

It means that because of rounding error approximation must be designed with smaller error so, that after considering both of these errors the approximation meets the requirement given by the value Δ_C. It can be shown that in our cases (NIP of degree 0, 1 and 2) is $H=1$ and approximation is projected with error

$$\Delta_{APR} = \Delta_C - \Delta_R \qquad (21)$$

The last error, which is considered, is error caused by AD conversion Δ'_{AD}. In the microcomputer there is realised nonlinear function. This function causes change of maximal error of AD conversion (quantization error) from Δ_A to Δ'_{AD} an for the relative error

$$\delta'_{AD} \ge \delta_{AD} \qquad (22)$$

Errors would be equal if a linear function is implemented. Process of error change is shown in the fig.7 (axis are labeled according to the fig.6). Somewhere inside of measurement scale error might be reduced but maximal error is always bigger due to the nonlinearity. It can be estimated by means of first derivation of approximated function.

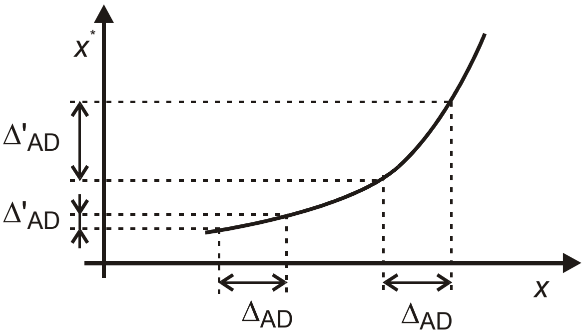

Fig.7 Influence of nonlinear function on the error of AD conversion

Resuming all mentioned errors it was found out that the sum of all errors influencing approximation in the microcomputer must be smaller than demanded error

$$\Delta_{APR} + \Delta_R + \Delta'_{AD} \leq \Delta_C \tag{23}$$

5. Realization

Function approximation was implemented into a microcomputer. Fig.8 shows NIP of degree 1 employed for sensor characteristic linearization. The points in this graph are the nodes and every line between two nodes is one polynomial. The scale of input N_x value indicates usage of 10-bit ADC.

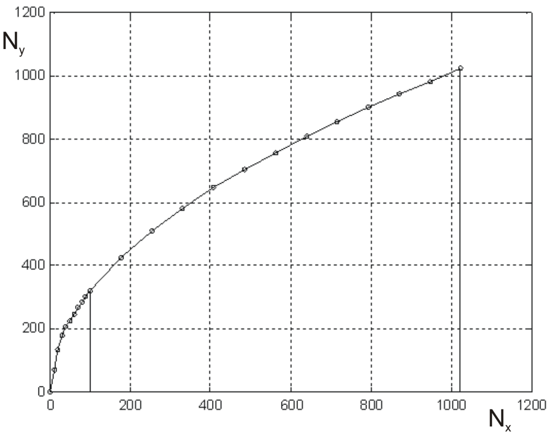

Fig.8 Inverse characteristics approximation using NIP of degree 1

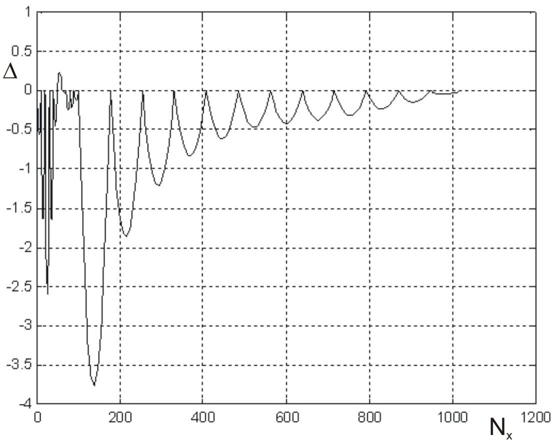

Fig.9 Error of approximation with NIP of degree 1

Dependence of approximation error from input value N_x is depicted in the fig.9. This could be drawn if the approximated function (original) is known.

In the microcomputer the value in each point is rounded, first in AD converter and then in output (DAC or display). The difference between mathematical approximation and function implemented in microcomputer could be seen in the fig.10. The error function forms "envelope" of the mathematical approximation error function, fig.11.

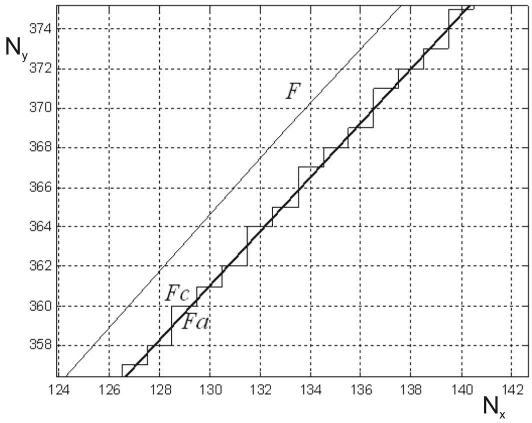

Fig.10 Comparison of mathematical and implemented approximation

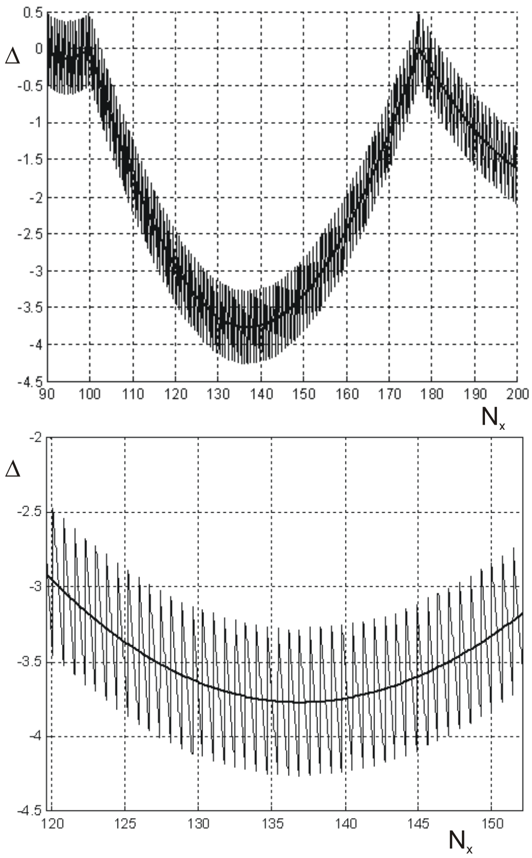

Fig.11 Error of approximation in the microcomputer in details

Conclusion

Approximation with Newton interpolation polynomial has been theoretically described. For approximation with polynomial in the microcomputer error analysis has been made. Three major error sources has been considered and included in the final equation (23) for approximation error. Understanding of error sources is the basic part for approximation design which is the main part of digital sensor characteristic linearization. The examples of errors from the approximation design have been demonstrated.

Acknowledgement

This work was supported by project VEGA 1/0153/03.

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