State constrained LQ control systems

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Abstract
The paper deals with the discrete-time LQ control problem for systems with state constrains, defined by linear equality. That control design can be viewed as a specific pole-assignment problem, where the equality constrains cause reduction of the allowable input space. These conditions are included in the modified Riccati equation to design controllers that control the closed-loop system in the optimal sense. In the paper some generalized considerations for the algorithm procedures are given and the problem of equivalent control is outlined.

Keywords: Discrete-time systems, LQ control methods, equality constraints.

Introduction

One of the major tasks in process control is maintain the process at the desired steady state operation level and accept variability of the process variables and so in industrial application it is desirable and sometimes necessary to consider constrains for controlled variables explicitly. In the traditional LQ control the joint variation of the output variables and the input variables is minimized using weighting matrices in performance index bat standard LQ controller design methods can not solve simultaneously problem of constrains and optimization of LQ control performance. In this sense another fundamental task in LQ control synthesis are constrains on system state/or input variables closely connected with system performance.

In the last years many significant results have spurred interest in problem of determining control laws for systems with constrains. One approach to the problem of finding the optimal results is technique dealing with system constrains directly. If this constrained problem is solvable, then one can modify optimized linear quadratic control performance index to adapt it for constrains. A special form of this constrained LQ problem can be formulated with the goal to optimize state feedback controller parameters while the system state variables satisfy the equality constrains.

In this paper one class of methods to solve the optimization problem concerning with discrete-time LQ control design for linear systems with state variable equality constrains is considered where presented result is an extension of methodology given in [2]. Based on the system state description, on given performance index and on the system state constrain equation the generalized performance index was obtained, which can be solved using standard form of Riccati equation for time-invariant discrete LQ control. Finally numerical example is shown in this paper to demonstrate the role of constrains in the optimization procedure.

1. LQ control task

The systems under consideration are discrete-time linear multi input/multi output (MIMO) dynamic systems

$$\mathbf{q}(i+1)=\mathbf{F}\mathbf{q}(i)+\mathbf{G}\mathbf{u}(i) \tag{1}$$

$$\mathbf{y}(i)=\mathbf{C}\mathbf{q}(i) \tag{2}$$

where $\mathbf{q}(i)\in\mathbb{R}^{n}$, $\mathbf{u}(i)\in\mathbb{R}^{r}$, $\mathbf{y}(i)\in\mathbb{R}^{m}$, respectively, and matrices $\mathbf{F}\in\mathbb{R}^{nxn}$, $\mathbf{G}\in\mathbb{R}^{nxr}$, and $\mathbf{C}\in\mathbb{R}^{mxn}$ are finite valued.

For such system (1), (2) the optimal control design task is, in general, to determine the control

$$\mathbf{u}(i)=-\mathbf{K}\mathbf{q}(i) \tag{3}$$

that minimizes the quadratic cost function

$$J_N=\mathbf{q}^T(N)\mathbf{Q}_N\mathbf{q}(N)+\sum_{i=0}^{N-1}s(\mathbf{q}(i),\mathbf{u}(i)) \tag{4}$$

$$\begin{gathered}s(\mathbf{q}(i),\mathbf{u}(i))=\\ =\mathbf{q}^T(i)\mathbf{Q}\mathbf{q}(i)+\mathbf{q}^T(i)\mathbf{S}\mathbf{u}(i)+\mathbf{u}^T(i)\mathbf{S}^T\mathbf{q}(i)+\mathbf{u}^T(i)\mathbf{R}\mathbf{u}(i)=\\ \begin{bmatrix}\mathbf{q}^T(i) & \mathbf{u}^T(i)\end{bmatrix}\begin{bmatrix}\mathbf{Q} & \mathbf{S}\\ \mathbf{S}^T & \mathbf{R}\end{bmatrix}\begin{bmatrix}\mathbf{q}(i)\\ \mathbf{u}(i)\end{bmatrix}=\begin{bmatrix}\mathbf{q}^T(i) & \mathbf{u}^T(i)\end{bmatrix}\mathbf{J}_J(i)\begin{bmatrix}\mathbf{q}(i)\\ \mathbf{u}(i)\end{bmatrix}\end{gathered} \tag{5}$$

$$\mathbf{Q}-\mathbf{S}\mathbf{R}^{-1}\mathbf{S}^T\geq 0 \tag{6}$$

where matrices $\mathbf{Q}=\mathbf{Q}^T\geq 0\in\mathbb{R}^{nxn}$, $\mathbf{Q}_N=\mathbf{Q}_N{}^T\geq 0\in\mathbb{R}^{nxn}$, and $\mathbf{R}=\mathbf{R}^T>0\in\mathbb{R}^{rxr}$ has full row rank, $\mathbf{S}=\in\mathbb{R}^{nxr}$ satisfies (6) and $\mathbf{K}=\in\mathbb{R}^{nxr}$ is the optimal control gain matrix.

2. Matrix pseudoinverse

Let $\mathbf{A}$, $\mathbf{B}$, $\mathbf{X}$, $\mathbf{Y}$ be matrices with consistent dimension satisfying equation

$$\mathbf{AXB}=\mathbf{Y} \tag{7}$$

Multiplying (7) by identity matrix from left hand side, as well as from right hand side one can obtain

$$\mathbf{AXB}=\mathbf{AA}^T(\mathbf{AA}^T)^{-1}\mathbf{Y}(\mathbf{B}^T\mathbf{B})^{-1}\mathbf{B}^T\mathbf{B} \tag{8}$$

$$\mathbf{X}=\mathbf{A}^{+}\mathbf{Y}\mathbf{B}^{+} \tag{9}$$

where

$$\mathbf{A}^{+}=\mathbf{A}^T(\mathbf{AA}^T)^{-1},\qquad \mathbf{B}^{+}=(\mathbf{B}^T\mathbf{B})^{-1}\mathbf{B}^T \tag{10}$$

is $\mathbf{A}$ and $\mathbf{B}$ **matrix pseudoinverse**.

Let $\mathbf{ZB}=\mathbf{C}$, then

$$\mathbf{Z}=\mathbf{CB}^{+}=\mathbf{ZBB}^{+} \tag{11}$$

$$\mathbf{Z}(\mathbf{I}-\mathbf{BB}^{+})=\mathbf{0} \tag{12}$$

and analogously for $\mathbf{D} = \mathbf{AX}$

$$\mathbf{Z} = \mathbf{A}^{+}\mathbf{D} = \mathbf{A}^{+}\mathbf{AZ} \tag{13}$$

$$(\mathbf{I} - \mathbf{A}^{+}\mathbf{A})\mathbf{Z} = \mathbf{0} \tag{14}$$

where matrix $\mathbf{Z}$ is an arbitrary matrix of appropriated dimension. Then

$$(\mathbf{I} - \mathbf{A}^{+}\mathbf{A})\mathbf{Z}(\mathbf{I} - \mathbf{BB}^{+}) = \mathbf{0} \tag{15}$$

$$\begin{aligned} &\mathbf{Z} + \mathbf{A}^{+}\mathbf{AZBB}^{+} - \mathbf{A}^{+}\mathbf{AZ} - \mathbf{ZBB}^{+} = \\ &= -\mathbf{Z} + \mathbf{A}^{+}\mathbf{AZBB}^{+} + (\mathbf{I} - \mathbf{A}^{+}\mathbf{A})\mathbf{Z} + \mathbf{Z}(\mathbf{I} - \mathbf{BB}^{+}) = \\ &-\mathbf{Z} + \mathbf{A}^{+}\mathbf{AZBB}^{+} = \mathbf{0} \end{aligned} \tag{16}$$

Using (9) and (16) all solution of (7) is

$$\mathbf{X} = \mathbf{A}^{+}\mathbf{YB}^{+} + \mathbf{Z} - \mathbf{A}^{+}\mathbf{AZBB}^{+} \tag{17}$$

3. Constrained control

Using control law (3) the steady-state equilibrium control equation takes the form

$$\mathbf{q}(i+1) = (\mathbf{F} - \mathbf{GK})\mathbf{q}(i) \tag{18}$$

$$\mathbf{y}(i) = \mathbf{Cq}(i) \tag{19}$$

Considering a design constrain

$$\mathbf{q}(i) \in \mathcal{Q} = \{\mathbf{q} : \mathbf{Dq} = \mathbf{0}\} \tag{20}$$

the state-variable vectors have to satisfy equalities

$$\mathbf{Dq}(i+1) = \mathbf{D}(\mathbf{F} - \mathbf{GK})\mathbf{q}(i) = \mathbf{0} \tag{21}$$

$$\mathbf{D}(\mathbf{F} - \mathbf{GK}) = \mathbf{0} \tag{22}$$

$$\mathbf{DF} = \mathbf{DGK} \tag{23}$$

respectively.

Using (17) all solutions of $\mathbf{K}$ are

$$\mathbf{K} = (\mathbf{DG})^{+}\mathbf{DF} + \mathbf{H} - (\mathbf{DG})^{+}\mathbf{DGH} \tag{24}$$

where $\mathbf{II}^{+} = \mathbf{I}$, $\mathbf{H}$ is an arbitrary matrix with appropriated dimension and

$$(\mathbf{DG})^{+} = (\mathbf{DG})^{T}(\mathbf{DG}(\mathbf{DG})^{T})^{-1} \tag{25}$$

is the pseudoinverse of $\mathbf{DG}$.

One can therefore express (24) as

$$\mathbf{K} = (\mathbf{DG})^{+}\mathbf{DF} + (\mathbf{I} - (\mathbf{DG})^{+}\mathbf{DG})\mathbf{H} \tag{26}$$

$$\mathbf{K} = \mathbf{M} + \mathbf{NH} \tag{27}$$

respectively, where

$$\mathbf{M} = (\mathbf{DG})^{+}\mathbf{DF} \tag{28}$$

$$\mathbf{N} = \mathbf{I} - (\mathbf{DG})^{+}\mathbf{DG} = \mathbf{I} - (\mathbf{DG})^{T}(\mathbf{DG}(\mathbf{DG})^{T})^{-1}\mathbf{DG} \tag{29}$$

is the projection matrix (the orthogonal projector onto the null space $\mathcal{N}(\mathbf{DG})$ of $\mathbf{DG}$). This result in

$$\mathbf{u}(i) = -\mathbf{Mq}(i) + \mathbf{N}(-\mathbf{Hq}(i)) = -\mathbf{Mq}(i) + \mathbf{N}\tilde{\mathbf{u}}(i) \tag{30}$$

$$\begin{bmatrix} \mathbf{q}(i) \\ \mathbf{u}(i) \end{bmatrix} = \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ -\mathbf{M} & \mathbf{N} \end{bmatrix} \begin{bmatrix} \mathbf{q}(i) \\ \tilde{\mathbf{u}}(i) \end{bmatrix} = \mathbf{T} \begin{bmatrix} \mathbf{q}(i) \\ \tilde{\mathbf{u}}(i) \end{bmatrix} \tag{31}$$

respectively, where

$$\mathbf{T} = \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ -\mathbf{M} & \mathbf{N} \end{bmatrix} \tag{32}$$

$$\tilde{\mathbf{u}}(i) = -\mathbf{Hq}(i) \tag{33}$$

4. Performance index

The best obtainable quadratic Lyapunov function is of the form

$$v(\mathbf{q}(i)) = \mathbf{q}^{T}(i)\mathbf{P}(i-1)\mathbf{q}(i) \tag{34}$$

where $\mathbf{P}(i\text{-}1) = \mathbf{P}(i\text{-}1)^{T} > 0 \in \mathbb{R}^{nxn}$ is a symmetric positive definite matrix and

$$\mathbf{P}(-1) = \mathbf{P}(0) \tag{35}$$

If Lyapunov function takes form (34), its difference is

$$\Delta v(\mathbf{q}(i), \mathbf{u}(i)) = v(\mathbf{q}(i+1)) - v(\mathbf{q}(i)) \tag{36}$$

$$\Delta v(\mathbf{q}(i), \mathbf{u}(i)) = \begin{bmatrix} \mathbf{q}^{T}(i) & \mathbf{u}^{T}(i) \end{bmatrix} \mathbf{J}_{V}(i) \begin{bmatrix} \mathbf{q}(i) \\ \mathbf{u}(i) \end{bmatrix} \tag{37}$$

respectively, where

$$\mathbf{J}_{V}(i) = \begin{bmatrix} \mathbf{F}^{T}\mathbf{P}(i)\mathbf{F} - \mathbf{P}(i-1) & \mathbf{F}^{T}\mathbf{P}(i)\mathbf{G} \\ (\mathbf{F}^{T}\mathbf{P}(i)\mathbf{G})^{T} & \mathbf{G}^{T}\mathbf{P}(i)\mathbf{G} \end{bmatrix} \tag{38}$$

and Lyapunov function at the time instant N takes value

$$V_{N-1} = \sum_{i=0}^{N-1} \Delta v(\mathbf{q}(i), \mathbf{u}(i)) \tag{39}$$

This, in turn, is equivalent with formula

$$V_{N-1} = \mathbf{q}^{T}(N)\mathbf{P}(N-1)\mathbf{q}(N) - \mathbf{q}^{T}(0)\mathbf{P}(0)\mathbf{q}(0) \tag{40}$$

Adding (39) and subtracting (40) to (4) the performance index for control law can be brought to the form

$$J_{N} = \mathbf{q}^{T}(0)\mathbf{P}(0)\mathbf{q}(0) + \sum_{i=0}^{N-1} p(\mathbf{q}(i), \mathbf{u}(i)) \tag{41}$$

where

$$\mathbf{P}(N-1) = \mathbf{Q}_{N} \tag{42}$$

$$\begin{aligned} \Delta v(\mathbf{q}(i), \mathbf{u}(i)) &= s(\mathbf{q}(i), \mathbf{u}(i)) + \Delta v(\mathbf{q}(i), \mathbf{u}(i)) = \\ &= \begin{bmatrix} \mathbf{q}^{T}(i) & \mathbf{u}^{T}(i) \end{bmatrix} \mathbf{J}(i) \begin{bmatrix} \mathbf{q}(i) \\ \mathbf{u}(i) \end{bmatrix} \end{aligned} \tag{43}$$

$$\mathbf{J}(i) = \mathbf{J}_{J}(i) + \mathbf{J}_{V}(i) = \begin{bmatrix} \mathbf{F}^{T}\mathbf{P}(i)\mathbf{F} - \mathbf{P}(i-1) + \mathbf{Q} & \mathbf{F}^{T}\mathbf{P}(i)\mathbf{G} + \mathbf{S} \\ (\mathbf{F}^{T}\mathbf{P}(i)\mathbf{G} + \mathbf{S})^{T} & \mathbf{G}^{T}\mathbf{P}(i)\mathbf{G} + \mathbf{R} \end{bmatrix} \tag{44}$$

With the new variable (33) the performance index (41) can be equivalently rewritten to the form

$$J_{N} = \mathbf{q}^{T}(0)\mathbf{P}(0)\mathbf{q}(0) + \sum_{i=0}^{N-1} p(\mathbf{q}(i), \tilde{\mathbf{u}}(i)) \tag{45}$$

where

$$p(\mathbf{q}(i), \tilde{\mathbf{u}}(i)) = \begin{bmatrix} \mathbf{q}^{T}(i) & \tilde{\mathbf{u}}^{T}(i) \end{bmatrix} \tilde{\mathbf{J}}(i) \begin{bmatrix} \mathbf{q}(i) \\ \tilde{\mathbf{u}}(i) \end{bmatrix} \tag{46}$$

$$\tilde{\mathbf{J}}(i) = \mathbf{T}^{T}\mathbf{J}(i)\mathbf{T} = \begin{bmatrix} \tilde{\mathbf{J}}_{11}(i) & \tilde{\mathbf{J}}_{12}(i) \\ \tilde{\mathbf{J}}_{12}^{T}(i) & \tilde{\mathbf{J}}_{22}(i) \end{bmatrix} \tag{47}$$

$$\begin{aligned} \tilde{\mathbf{J}}_{11}(i) = &\mathbf{F}^{T}\mathbf{P}(i)\mathbf{F} - \mathbf{P}(i-1) + \mathbf{Q} - (\mathbf{F}^{T}\mathbf{P}(i)\mathbf{G} + \mathbf{S})\mathbf{M} - \\ &-\mathbf{M}^{T}(\mathbf{F}^{T}\mathbf{P}(i)\mathbf{G} + \mathbf{S})^{T} + \mathbf{M}^{T}(\mathbf{G}^{T}\mathbf{P}(i)\mathbf{G} + \mathbf{R})\mathbf{M} \end{aligned} \tag{48}$$

$$\tilde{\mathbf{J}}_{12}(i) = (\mathbf{F}^{T}\mathbf{P}(i)\mathbf{G} + \mathbf{S})\mathbf{N} - \mathbf{M}^{T}(\mathbf{G}^{T}\mathbf{P}(i)\mathbf{G} + \mathbf{R})\mathbf{N} \tag{49}$$

$$\tilde{\mathbf{J}}_{22}(i) = \mathbf{N}^{T}(\mathbf{G}^{T}\mathbf{P}(i)\mathbf{G} + \mathbf{R})\mathbf{N} \tag{50}$$

Thus, an equivalent standard form of (47), i.e. form

$$\tilde{\mathbf{J}}(i)=\begin{bmatrix}\mathbf{F}^{\circ T}\mathbf{P}(i)\mathbf{F}^{\circ}-\mathbf{P}(i-1)+\mathbf{Q}^{\circ} & \mathbf{F}^{\circ T}\mathbf{P}(i)\mathbf{G}^{\circ}+\mathbf{S}^{\circ}\\ (\mathbf{F}^{\circ T}\mathbf{P}(i)\mathbf{G}^{\circ}+\mathbf{S}^{\circ})^{T} & \mathbf{G}^{\circ T}\mathbf{P}(i)\mathbf{G}^{\circ}+\mathbf{R}^{\circ}\end{bmatrix} \quad (51)$$

can be obtained using notations

$$\mathbf{F}^{\circ}=\mathbf{F}-\mathbf{GM} \quad (52)$$

$$\mathbf{G}^{\circ}=\mathbf{GN} \quad (53)$$

$$\mathbf{Q}^{\circ}=\mathbf{Q}+\mathbf{M}^{T}\mathbf{RM}-\mathbf{SM}-\mathbf{M}^{T}\mathbf{S}^{T} \quad (54)$$

$$\mathbf{R}^{\circ}=\mathbf{N}^{T}\mathbf{RN} \quad (55)$$

$$\mathbf{S}^{\circ}=(\mathbf{S}-\mathbf{M}^{T}\mathbf{R})\mathbf{N} \quad (56)$$

5. Control optimization

Accepting all above given notations there exist such optimal control satisfying conditions

$$\frac{\partial p(\mathbf{q}(i),\tilde{\mathbf{u}}(i))}{\partial\tilde{\mathbf{u}}^{T}(i)}=\begin{bmatrix}\mathbf{0}^{T} & \mathbf{I}\end{bmatrix}\tilde{\mathbf{J}}(i)\begin{bmatrix}\mathbf{q}(i)\\ \tilde{\mathbf{u}}(i)\end{bmatrix}=$$
$$=\begin{bmatrix}(\mathbf{F}^{\circ T}\mathbf{P}(i)\mathbf{G}^{\circ}+\mathbf{S}^{\circ})^{T} & \mathbf{G}^{\circ T}\mathbf{P}(i)\mathbf{G}^{\circ}+\mathbf{R}^{\circ}\end{bmatrix}\begin{bmatrix}\mathbf{q}(i)\\ \tilde{\mathbf{u}}(i)\end{bmatrix}=\mathbf{0} \quad (57)$$

$$\frac{\partial p(\mathbf{q}(i),\tilde{\mathbf{u}}(i))}{\partial\mathbf{q}^{T}(i)}=\begin{bmatrix}\mathbf{I} & \mathbf{0}^{T}\end{bmatrix}\tilde{\mathbf{J}}(i)\begin{bmatrix}\mathbf{q}(i)\\ \tilde{\mathbf{u}}(i)\end{bmatrix}= \quad (58)$$
$$=\begin{bmatrix}\mathbf{F}^{\circ T}\mathbf{P}(i)\mathbf{F}^{\circ}-\mathbf{P}(i-1)+\mathbf{Q}^{\circ} & \mathbf{F}^{\circ T}\mathbf{P}(i)\mathbf{G}^{\circ}+\mathbf{S}^{\circ}\end{bmatrix}\begin{bmatrix}\mathbf{q}(i)\\ \tilde{\mathbf{u}}(i)\end{bmatrix}=\mathbf{0}$$

Therefore, the vector variable (33), i.e. vector

$$\tilde{\mathbf{u}}(i)=-\mathbf{H}(i)\mathbf{q}(i) \quad (59)$$

can be computed using (57), where

$$\mathbf{H}(i)=(\mathbf{G}^{\circ T}\mathbf{P}(i)\mathbf{G}^{\circ}+\mathbf{R}^{\circ})^{-1}(\mathbf{F}^{\circ T}\mathbf{P}(i)\mathbf{G}^{\circ}+\mathbf{S}^{\circ})^{T} \quad (60)$$

P = P^{T} > 0 is a solution of discrete Riccati equation

$$\mathbf{P}(i-1)=\mathbf{F}^{\circ T}\mathbf{P}(i)\mathbf{F}^{\circ}+\mathbf{Q}^{\circ}-(\mathbf{F}^{\circ T}\mathbf{P}(i)\mathbf{G}^{\circ}+\mathbf{S}^{\circ})\mathbf{H}(i) \quad (61)$$

and resulting solution to the LQ problem with state equality constrains is given by the optimal control law

$$\mathbf{u}(i)=-\mathbf{Mq}(i)+\mathbf{N}\tilde{\mathbf{u}}(i)=-(\mathbf{M}+\mathbf{NH}(i))\mathbf{q}(i) \quad (62)$$

The constant gain state feedback controller for infinity control time and control law (30) takes form

$$\mathbf{u}(i)=-(\mathbf{M}+\mathbf{NH})\mathbf{q}(i)=-\mathbf{Kq}(i) \quad (63)$$

$$\mathbf{H}=(\mathbf{G}^{\circ T}\mathbf{PG}^{\circ}+\mathbf{R}^{\circ})^{-1}(\mathbf{F}^{\circ T}\mathbf{PG}^{\circ}+\mathbf{S}^{\circ})^{T} \quad (64)$$

and is given by a steady-state solution **P** of (61), i.e. by the solution of the algebraic Riccati equation

$$\mathbf{P}=\mathbf{F}^{\circ T}\mathbf{P}(i)\mathbf{F}^{\circ}+\mathbf{Q}^{\circ}-$$
$$-(\mathbf{F}^{\circ T}\mathbf{PG}^{\circ}+\mathbf{S}^{\circ})(\mathbf{G}^{\circ T}\mathbf{PG}^{\circ}+\mathbf{R}^{\circ})^{-1}(\mathbf{F}^{\circ T}\mathbf{PG}^{\circ}+\mathbf{S}^{\circ})^{T} \quad (65)$$

6. Illustrative example

Consider the plat with two-inputs and two-outputs, described by discrete-time model with sampling period Δt = 0.1s and matrix parameters

$$\mathbf{F}=\begin{bmatrix}0.9993 & 0.0987 & 0.0042\\ 0.0212 & 0.9612 & 0.0775\\ 0.3985 & 0.7187 & 0.5737\end{bmatrix}$$

$$\mathbf{G}=\begin{bmatrix}0.0051 & 0.0050\\ 0.1029 & 0.9612\\ 0.0387 & 0.5737\end{bmatrix},\quad \mathbf{C}=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 1\end{bmatrix}$$

Assuming the performance index (4), weighting matrices

$$\mathbf{Q}=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix},\quad \mathbf{R}=0.01\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix},\quad \mathbf{S}=0.01\begin{bmatrix}0 & 0\\ 1 & 1\\ 0 & 0\end{bmatrix}$$

and the design constrains

$$\mathbf{D}=\begin{bmatrix}2 & -1 & -1\end{bmatrix}$$

there were obtained feedback gain matrix parameters

$$\mathbf{M}=\begin{bmatrix}-10.5747 & 9.8603 & 4.2754\\ -4.0158 & 3.7455 & 1.6236\end{bmatrix},\quad \mathbf{N}=\begin{bmatrix}0.1260 & -0.3319\\ -0.3319 & 0.8740\end{bmatrix}$$

New design parameters were then recomputed as follows

$$\mathbf{F}^{\circ}=\begin{bmatrix}1.0733 & 0.0297 & -0.0257\\ 1.5057 & -0.4230 & -0.5227\\ 0.6409 & 0.4824 & 0.4712\end{bmatrix},\quad \mathbf{G}^{\circ}=\begin{bmatrix}-0.0010 & 0.0027\\ -0.0198 & 0.0521\\ 0.0178 & -0.0468\end{bmatrix}$$

$$\mathbf{Q}^{\circ}=\begin{bmatrix}2.2795 & -1.3390 & -0.5173\\ -1.3390 & 2.3846 & 0.5414\\ -0.5173 & 0.5414 & 1.2091\end{bmatrix}$$

$$\mathbf{R}=0.01\begin{bmatrix}0.13 & -0.33\\ -0.33 & 0.87\end{bmatrix},\quad \mathbf{S}=0.01\begin{bmatrix}0.00 & 0.00\\ -0.21 & 0.54\\ 0.00 & 0.00\end{bmatrix}$$

Applying the Matlab function *dare*(.) to design matrix **H,** the optimal solution was obtained as

$$\mathbf{H}=\begin{bmatrix}-95.8183 & 28.0602 & 13.2447\\ -0.0000 & 8.0000 & 0.0000\end{bmatrix}$$

and final feedback gain matrix was

$$\mathbf{K}=\mathbf{M}+\mathbf{NH}=\begin{bmatrix}-22.6515 & 10.7419 & 5.9447\\ 27.7856 & 1.4232 & -2.7722\end{bmatrix}$$

One can easily verify, that closed-loop system matrix

$$\mathbf{F}_{c}=\mathbf{F}-\mathbf{GK}=\begin{bmatrix}0.9759 & 0.0368 & -0.0123\\ -0.3904 & -0.2846 & -0.2606\\ 2.3422 & 0.3582 & 0.2361\end{bmatrix}$$

is stabile matrix with eigenvalues spectrum

$$\rho(\mathbf{F}_{c})=\{0.0000, 0.0242, 0.9031\}$$

and design constrain

$$\mathbf{DF}_{c}\doteq\begin{bmatrix}0 & 0 & 0\end{bmatrix}$$

Concluding remarks

Based on the state-space equation, the performance index parameters and system constrain for time-invariant discrete LQ control problem, the generalized Riccati equations of linear equality constraint system is presented and finally numerical example is shown in this paper. The proposed method presents some new design features and generalizations. It should be emphasized that the advantage offered by the approach is in its computational simplicity.

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