Stabilizing Predictive Control of Fuzzy Systems Described by Takagi-Sugeno Models

Martin Herceg, Michal Kvasnica, Miroslav Fikar

Abstract
The paper concerns with synthesis of stabilizing controllers for processes described by fuzzy Takagi-Sugeno models. Commonly in the fuzzy control it is difficult to design control strategies which guarantee that the closed-loop system will be asymptotically stable. Standard approaches which rely on co-stabilization of the whole collection of local models are usually overly conservative. Therefore we propose to transform the fuzzy description into a piecewise affine (PWA) model for which efficient control strategies guaranteeing closed-loop stability exist. The PWA transformation procedure approximates the overlaps naturally present in fuzzy models by means of an unknown, but bounded additive uncertainty. Once such a PWA model is available, we propose to use a minimum-time strategy to design a stabilizing feedback law. Moreover, we show that the controller takes a form of a look-up table, which can be evaluated in real-time.

Key words Takagi-Sugeno models, Piecewise Affine models, Model Predictive Control

Introduction

The main aim of this paper is synthesis of stabilizing feedback laws for systems described by fuzzy models. It has been shown that fuzzy modelling can approximate any process with prescribed accuracy and therefore it can be classified as an universal approximation [12]. Although the modeling issues are not addressed in this paper some valuable references can be found for example in [5] or [1].

Despite the attractiveness of fuzzy models, little is known about how to derive controllers guaranteeing closed-loop stability for such models. Traditional approaches solve a co-stabilization problem by means of linear matrix inequalities [15], [22], [11]. These techniques are, however, overly conservative, since they assume that all possible local dynamical models are all active at the same time. The problems gets more complicated if fulfillment of state and input constraints has to be guaranteed. Therefore the concept of Model Predictive Control (MPC) has been adopted to the class of fuzzy models. MPC is an optimization-based control policy widely adopted by the industry due to its ability to provide optimal performance together with constraint satisfaction [17]. In the MPC framework, the prior knowledge of the process behavior, represented by the prediction model, is used to design a sequence of control inputs such that certain performance criterion is optimized. Contrary to classical proportional-integral-derivative (PID) controllers, the decisions are done with respect to process properties and constraints.

Depending on the model used, slightly different approaches were developed. Supposedly the first approach was made by impulse response models, pioneered by [7] and [18], denoted as Dynamic Matrix Control (DMC). Revolutionary contribution was brought by [6] where the step responses serve for predictions, often abbreviated as Generalized Predictive Control (GPC). The growing need for tighter controlling demands motivated use of nonlinear models. This novel approach is nowadays referred as Nonlinear Model Predictive Control (NMPC) [2], [13]. Recently, many predictive strategies which employ fuzzy models emerged and this class of control problems is referred to as Fuzzy Model Predictive Control (FMPC).

An excellent comparative study [8] provides a deeper view into four recently developed predictive strategies for fuzzy systems. Each of the strategy uses the GPC approach but the control action is calculated differently. It is either a linear combination of all locally designed controllers or a global controller acting over a linear time-varying model (LTV). A hierarchical structure of multiple Takagi-Sugeno models, proposed by [10] and [16], deploys also GPC approach while the controller is obtained by weighted aggregation over governing local rules. All of the aforementioned approaches, however, one common property, namely they do not discuss issues regarding closed-loop stability in presence of constraints.

In this paper we propose a different way of assuring stability and feasibility guarantees for closed-loop systems based on fuzzy Takagi-Sugeno (TS) models [21]. Motivated by the recent advances in the field of hybrid systems [3], we suggest to over-approximate a given Takagi-Sugeno fuzzy model by a Piecewise Affine (PWA) model, for which efficient control strategies ensuring closed-loop stability and infinite-time feasibility exist [19], [9]. Unlike TS fuzzy models, the PWA description requires that the regions, over which individual dynamics are defined, to be non-overlapping. Therefore we propose to approximate the overlaps naturally present in TS models by means of an unknown, but bounded additive uncertainty. The main contribution of the paper is represented by a constructive procedure to derive a PWA model from a Takagi-Sugeno fuzzy model. Once such a PWA model is derived, we then apply the minimum-time principle of [19] combined with a so-called *parametric* MPC framework [4] to obtain an MPC controller which guarantees closed-loop stability. Since the underlying mathematical model is an over-approximation of the original Takagi-Sugeno model, if such a controller exists, it will guarantee stability of the original model as well.

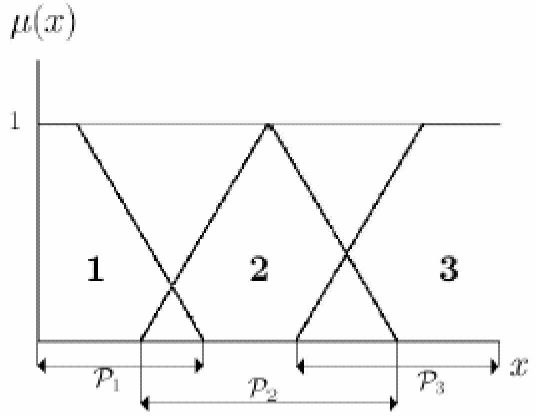

Fig. 1 Illustration of linear membership functions for three rules in the Takagi-Sugeno modelling approach.

Takagi-Sugeno Fuzzy Model Representation

The class of Takagi-Sugeno (TS) models can be generally described by fuzzy „IF ... THEN“ rules where the fuzzy sets stay on the antendecent side while the consequence is given by a linear dynamics. Generally, the i th TS rule can be expressed as

$$\begin{array}{ll} \text{IF} & x_{1,k} \text{ is } \mu_{i1} \text{ and } \ldots x_{n,k} \text{ is } \mu_{in} \\ \text{THEN} & x_{k+1} = A_i x_k + B_i u_k \end{array} \quad (1)$$

where $x_k \in \mathbf{R}^n$ is the state vector, $u_k \in \mathbf{R}^m$ denotes the vector of manipulated variables, μ_{ij} are input fuzzy sets for $i = 1,\ldots,r$ rules. $A_i \in \mathbf{R}^{n\times n}$, $B_i \in \mathbf{R}^{n\times m}$ are matrices representing the system dynamics. The process dynamics is assumed to be discretized with k denoting one sampling instant. The aggregated system output is modelled using the max-product inference, i.e.

$$x_{k+1} = \frac{\sum_{i=1}^{r} w_i(x_k)(A_i x_k + B_i u_k)}{\sum_{i=1}^{r} w_i(x_k)} \quad (2)$$

with

$$w_i(x_k) = \prod_{j=1}^{n} \mu_{ij}(x_{j,k}) \quad (3)$$

where the membership function μ_{ij} measures the activation of the fuzzy set j in the rule i. Using the notation

$$\alpha_i(x_k) = \frac{w_i(x_k)}{\sum_{i=1}^{r} w_i(x_k)}, \quad \sum_{i=1}^{r} \alpha_i(x_k) = 1 \quad (4)$$

the overall system model can be described as

$$x_{k+1} = \sum_{i=1}^{r} w_i(x_k)(A_i x_k + B_i u_k) \quad (5)$$

A simple TS model is illustrated in Fig. 1 using three rules with linear fuzzy membership functions. It can be seen that each dynamics contributes to the overall model with its corresponding membership function and moreover, if the state belongs to a region where more than one dynamics become active, then the weighted contribution of overlapping nodes is considered.

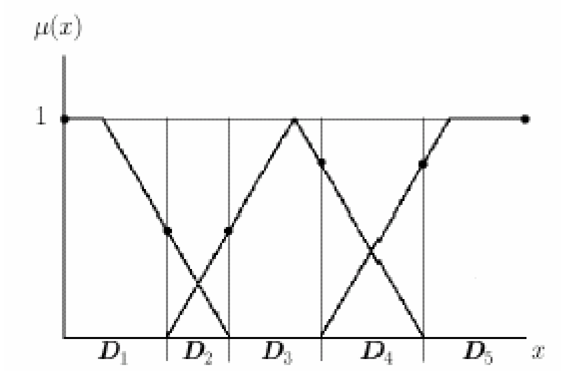

Fig. 2 Intersections of fuzzy sets are replaced by new regions with crisp boundaries.

The Transformation Procedure

In this section the main result of the paper will be presented. Consider the TS model (1) with linear fuzzy membership functions. The fuzzy input sets μ_{ij} can be decomposed in the following manner:

$$\text{IF } x_k \in P_i \text{ THEN } x_{k+1} = A_i x_k + B_i u_k \quad (6)$$

where the region P_i in which the corresponding rule is active can be described by a polyhedral set

$$P_i := \{H_i x_k \le K_i\} \quad (7)$$

The aim is to transform the TS model into a Piecewise Affine model of the following form:

$$\begin{aligned} x_{k+1} &= f_{PWA}(x_k, u_k, w_k) \\ &= A_i x_k + B_i u_k + f_i + w_k \quad \text{if} \quad x_k \in D_i \end{aligned} \quad (8)$$

with $A_i \in \mathbf{R}^{n\times n}$, $B_i \in \mathbf{R}^{n\times m}$, and $f_i \in \mathbf{R}^n$. Here, $\{D_i\}_{i=1}^{n_D} \in \mathbf{R}^n$ denotes a polyhedral partition satisfying $D = \bigcup_{i=1}^{n_D} D_i$. The measured state is denoted by x_k, manipulated inputs correspond to u_k, and w_k denotes an unknown additive disturbance. The system states x_k, control inputs u_k as well as the disturbance w_k of the system (8) are subject to the constraints

$$x_k \in X \subseteq \mathbf{R}^n, \; u_k \in U \subseteq \mathbf{R}^m, \; w_k \in W \subseteq \mathbf{R}^n \quad (9)$$

where X, U, and W are polyhedral sets containing the origin in their respective interiors.

To obtain the strictly separated regions D_i, the overlaps in the membership functions of the TS model have to be removed first. This can be done in a straightforward manner by defining new regions for each intersection of the neighboring fuzzy sets, i.e.

$$D_j = P_i \cap P_{i+1} \qquad j = 1,\dots,n_P \tag{10}$$

which is also a polyhedral set. If the set D_j is a subset of the next set (e.g. when more than 2 fuzzy sets intersect) then the statement

$$D_j \subset D_{j+1} \Rightarrow D_j = 0 \tag{11}$$

implies that the redundant sets will be removed. Fig. 2 depicts the decomposition of the fuzzy sets to a crisp sets by introducing additional regions D_2 and D_4, respectively. Because the regions P_i are represented by convex polytopes as in (7), the overall calculation of intersections can be performed using standard algebraic manipulation techniques.

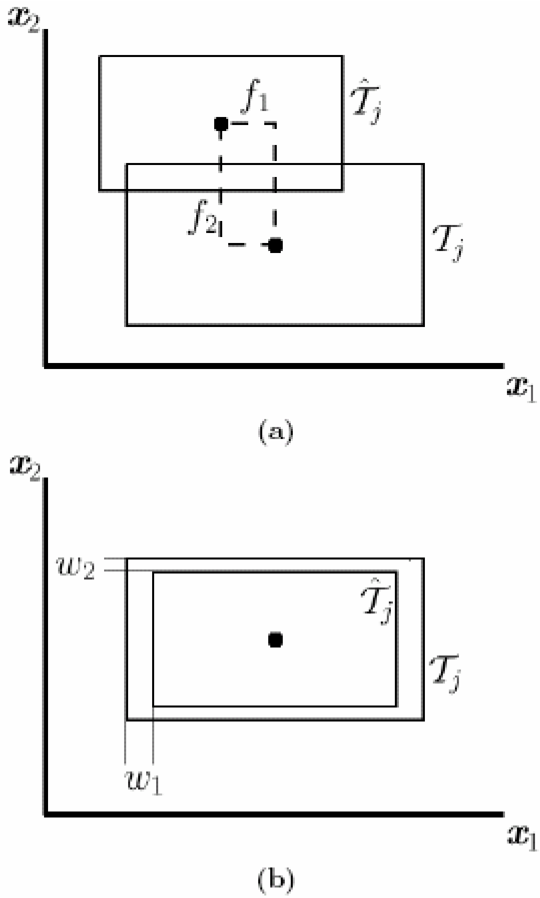

Fig. 3 The transformation procedure shifts the reachable sets to one common centre.

The remaining regions can be obtained by a set-difference operation

$$\begin{aligned} D_j &= \bigcup_i P_i \setminus \bigcup_{i=1}^{n_P} D_i \\ &= \left\{ x_k \mid x_k \in \bigcup_i P_i,\ x_k \notin \bigcup_{i=1}^{n_P} D_i \right\} \end{aligned} \tag{12}$$

Secondly, it is important to determine the mean PWA description for each region with bounded additive uncertainty, i.e. to express the transition from (2) to (8). To do so, the worst case perturbations of the mean model have to be considered. Obviously, these values will be located at the boundary of each region, as indicated by black dots in Fig. 2. Thus, the mean model for region D_j is given by arithmetic mean of neighboring models corresponding to the boundary of a given region, i.e.

$$\hat{A}_j = \frac{1}{n_n} \sum_i A_i, \qquad \hat{B}_j = \frac{1}{n_n} \sum_i B_i \tag{13}$$

with $i \in I_j$ where I_j stands for the index set of dynamics active in the region D_j and n_n denotes the number of overlapping models.

The next step is to determine the affine term f_j and the maximal allowed uncertainty w_j in each region D_j. For this purpose the maximum allowed reachable set of the uncertain system is explored. Let A_j, B_j denote the families of possible realizations of matrices $\hat{A}_j$, $\hat{B}_j$. An over-approximation of the maximum reachable set for the region D_j is given by

$$T_j := \{ x_{k+1} \mid \underline{x}_{k+1} \le x_{k+1} \le \overline{x}_{k+1} \} \tag{14}$$

where the update x_{k+1} of the state is driven by the TS model (5) and $\underline{x}_{k+1}$ and $\overline{x}_{k+1}$ denote, respectively, the lower and upper limits of all possible realizations of x_{k+1}. The key idea is to use an approximation of the form

$$\begin{aligned} x_{k+1} &\cong \hat{x}_{k+1} \\ \sum_{i=1}^{r} \alpha_i(x_k)(A_i x_k + B_i u_k) &\cong \hat{A}_j x_k + \hat{B}_j u_k \\ &\quad \text{if} \quad x_k \in D_j \subset X \end{aligned} \tag{15}$$

and to transform the model (15) into a PWA system with bounded additive disturbances (8). Note that the PWA model (8) actually over-approximates the behavior of the original problem (5) because even if the linearization for the particular regions D_j is determined, the conservatism appears in the unknown signal w where the maximum allowed disturbance is considered. Obviously, the transformation will be applied to regions where multiple membership functions overlap. In the remaining regions only a single dynamical model will be active.

Obtaining the maximum reachable set T_j for the sector D_j via solving (15) can be viewed as a collection of polytope operations. Define the partial reachable set for the model i in the region D_j by

$$Q_{ij} := \{ x_{k+1} \mid x_{k+1} = A_i x_k + B_i u_k,\ x_k \in D_j,\ u_k \in U \} \tag{16}$$

Consequently, the maximum reachable set for the region D_j can be found as the bounding box of the union of the partial sets, i.e.

$$T_j := \mathrm{Bbox}\Big(\bigcup_{i=1}^{n_n} Q_{ij}\Big) \tag{17}$$

where the operator Bbox is defined as follows:

Definition 1. [20] *A bounding box* $\mathrm{Bbox}(P)$ *of a set* P *is the smallest hyper-rectangle which contains the set* P. *If* P *is defined as a (possibly) non-convex union of convex polytopes* P_i, *i.e.* $P = \bigcup_i P_i$, *then the bounding box can be computed by solving* $2n$ *linear programs per each element of the set* P. *Here,* n *denotes the dimension of* P.

The maximum estimated reachable set $\hat{T}_j$ can be computed similarly as a bounding box of the reachable sets for the mean model (8):

$$\hat{T}_j = \mathrm{Bbox}(Q_j) \tag{18}$$

with

$$\hat{Q}_j := \{x_{k+1} \mid x_{k+1} = \hat{A}_j x_k + \hat{B}_j u_k,\ x_k \in D_j,\ u_k \in U\} \tag{19}$$

The affine terms f_j of (8) can now be computed as a difference between the analytic centers of the reachable sets for the “true” and for the “approximated” system:

$$f_j = \mathrm{ce}(T_j) - \mathrm{ce}(\hat{T}_j) \tag{20}$$

where the operator ce is given by

$$\mathrm{ce}(T) = \overline{x} - \frac{\overline{x} - \underline{x}}{2}. \tag{21}$$

Graphically are these sets depicted in Fig. 3a. It can be seen in Fig. 3b that the transformation procedure shifts these sets to one common analytic center. The allowable disturbance is then selected as the maximum distance over the edges of the sets in the sector D_j, i.e.

$$w_j = \begin{cases} \max(T_j - (\hat{T}_j + f_j)) & \text{if } T_j \geq \hat{T}_j + f_j \\ 0 & \text{otherwise} \end{cases} \tag{22}$$

In other words, if the approximated reachable set $\hat{T}_j$, shifted by the offset f_j, is smaller than the original reachable set T_j, then the difference is modeled by an unknown, but bounded disturbance w_j, whose element-wise bounds are given by (22).

By applying the same procedure to each sector D_j the original fuzzy model (2) can be converted to a PWA description (8). In the next section the pattern will be demonstrated on an illustrative example.

Stabilizing Control of Uncertain PWA System

In this section we review the algorithm to obtain a state-feedback control law which stabilizes an uncertain PWA system given by (8) with constraints of the form (9). In order to address the stabilization problem the following definition is needed.

Definition 2. [19] *The set $\psi \in X$ is said to be robustly control invariant for the PWA system (8) subject to the constraints (9) if for every $x_k \in \psi$ exist $u_k \in U$ such that $f_{PWA}(x_k, u_k, w_k) \in \psi,\ \forall w_k \in W$.*

Hence the objective of the control synthesis becomes to find a mapping $u(x_k) : \mathbf{R}^n \mapsto \mathbf{R}^m$ which can stabilize the uncertain system (8) in the sense of Definition 1. This problem was resolved in [19] using a minimum-time approach and in the sequel the overall scheme will be presented. The procedure consists primarily of two steps. First, a suitable target set is designed and a corresponding Lyapunov function is constructed to confirm that the control laws in the target set are stabilizing. Subsequently the target set is used to initialize an iterative algorithm which constructs the feedback law in a way such that all system states are forced to move into the stabilizing target set in a finite number of steps while respecting system constraints. The nature of the algorithm actually guarantees that the number of steps is minimal, hence the procedure is referred to as minimum-time control.

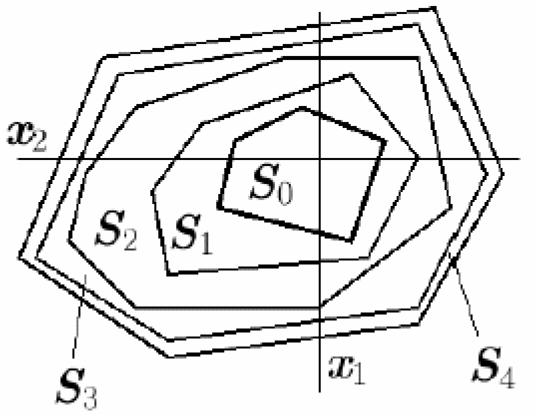

Fig. 4 Robusts invariant sets generated for the uncertain system (8). The algorithm evolves from the target set S_0 outwards and the union of the sets is the maximum robust stabilizable domain.

As already mentioned, the algorithm operates in an iterative fashion. At each iteration a Constrained Finite Time Optimal Control (CFTOC) problem of the following form is solved:

$$\begin{aligned} \min\quad & u_k^T R u_k + x_k^T Q x_k + x_{k+1}^T P x_{k+1} \\ \text{s.t.}\quad & x_{k+1} = f_{PWA}(x_k, u_k, w_k) \\ & x_{k+1} \in T_{set} \\ & u_k \in U \\ & w_k \in W \end{aligned} \tag{23}$$

with the terminal set constraint T_{set}. It is well known that if the value of x_k is fixed, the problem (23) can be casted as a standard Mixed Integer Quadratic Program (MIQP). However the solution to the optimization setup (23) can also be obtained in an explicit form using the so-called parametric programming techniques.

Theorem 1. *(Solution to CFTOC, [4]) The solution to the optimal control problem (23) is a piecewise affine state feedback optimal control law of the form*

$$u^*(x_k) = F_r^k x_k + G_r^k \quad \text{if } x_k \in P_r^k \tag{24}$$

where $P_r^k = \{x_k \in R^n \mid H_r^k x_k \leq K_r^k\},\ r = 1,\ldots,R^k$ is a polyhedral partition of the set X^k of feasible states x_k at time k. Here, R^k denotes the number of regions of the partition P^k.

One important implication of Theorem 1 is that the set of states P_r for which the problem (23) is feasible can be represented as a union of convex polytopes. This allows one to formulate the minimum-time algorithm as follows:

The Minimum-Time Algorithm

1. Calculate an initial stabilizing target set using reachability techniques as discussed by [19] Denote the set by S_0. Set the iteration counter $i = 0$.
2. Solve parametrically the optimization problem (23) with the terminal set $T_{set} = S_i$ constraint. Denote the feasible set of the solution to (23) by S_{i+1}.
3. If $S_{i+1} = S_i$, abort, the algorithm has converged

4. Otherwise increment the iteration counter $i = i+1$ and jump back to Step 2.

The first 4 iterations of such algorithm are visually depicted in Fig. 4.

According to Theorem 1, at each iteration of Step 2 we obtain a set of convex regions with an associated state-feedback law. The issue of real-time implementation of such control laws then reduces to a simple set-membership test which can be performed much more efficiently compared to a solution to MIQP problems. If the feedback laws associated to the initial terminal set stabilize a given unknown PWA system, the algorithm generates a control law which guarantees closed-loop stability [19]. For details regarding on-line implementation of minimum-time controllers we refer the reader to [9].

Example

Consider a TS model (1) described by two linear dynamics

$$A_1 = \begin{pmatrix} 0.2353 & 0.5457 \\ -0.4205 & 0.9492 \end{pmatrix},\ B_1 = \begin{pmatrix} 0.4416 \\ 0.7186 \end{pmatrix}$$
$$A_2 = \begin{pmatrix} -0.5226 & -0.7553 \\ -0.5920 & -0.4331 \end{pmatrix},\ B_1 = \begin{pmatrix} 0.8774 \\ 0.6517 \end{pmatrix} \tag{25}$$

associated with the following membership functions

$$\mu_1(x_1) = \begin{cases} 0 & \text{if } |x_1| \geq 1.5 \\ 1 - \frac{2}{3}|x_1| & \text{otherwise} \end{cases}$$
$$\mu_2(x_1) = \begin{cases} 0 & \text{if } |x_1| \leq 1.0 \\ -\frac{1}{2} + \frac{1}{2}|x_1| & \text{otherwise} \end{cases} \tag{26}$$

The functions are depicted in Fig. 5. Using the eigenvalue analysis it can be shown that the matrix A_1 represents a stable system, while A_2 has unstable modes.

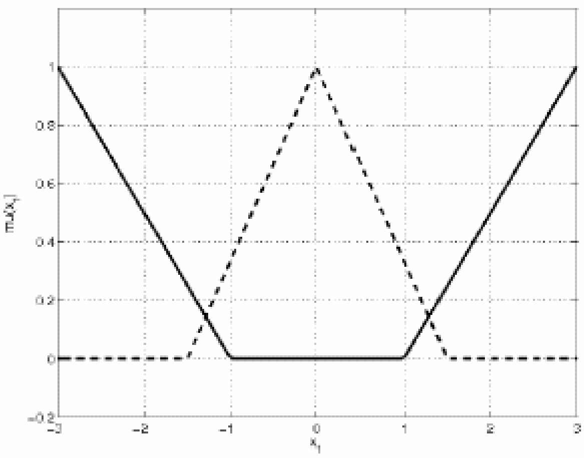

Fig. 5 Membership functions.

Constraints imposed for this example are the closed intervals

$$u_k \in [-5,\ 5], \qquad x_k \in [-3,\ 3] \times [-2,\ 2]. \tag{27}$$

To convert a given TS model into the PWA form (8), the feasible region (25) is first decomposed into 5 intervals given by following polytopes:

$$\begin{aligned} D_1 &:= \begin{pmatrix} -1 & 0 \\ 1 & 0 \end{pmatrix} x_k \leq \begin{pmatrix} 3 \\ -1.5 \end{pmatrix} \\ D_2 &:= \begin{pmatrix} -1 & 0 \\ 1 & 0 \end{pmatrix} x_k \leq \begin{pmatrix} 1.5 \\ -1 \end{pmatrix} \\ D_3 &:= \begin{pmatrix} -1 & 0 \\ 1 & 0 \end{pmatrix} x_k \leq \begin{pmatrix} 1 \\ 1 \end{pmatrix} \\ D_4 &:= \begin{pmatrix} -1 & 0 \\ 1 & 0 \end{pmatrix} x_k \leq \begin{pmatrix} -1 \\ 1.5 \end{pmatrix} \\ D_5 &:= \begin{pmatrix} -1 & 0 \\ 1 & 0 \end{pmatrix} x_k \leq \begin{pmatrix} -1.5 \\ 3 \end{pmatrix} \end{aligned} \tag{28}$$

The polytopes (26) have been selected following the procedure illustrated in Fig. 2. The PWA model takes the affine form

$$x_{k+1} = \begin{cases} A_2 x_k + B_2 u_k + w_k & \text{if } x_k \in D_1 \\ \hat{A}_{1,2} x_k + \hat{B}_{1,2} u_k + f_2 + w_k & \text{if } x_k \in D_2 \\ A_1 x_k + B_1 u_k + w_k & \text{if } x_k \in D_3 \\ \hat{A}_{1,2} x_k + \hat{B}_{1,2} u_k + f_2 + w_k & \text{if } x_k \in D_4 \\ A_2 x_k + B_2 u_k + w_k & \text{if } x_k \in D_5 \end{cases} \tag{29}$$

with $\hat{A}_{1,2} = 0.5A_1 + 0.5A_2$ and $\hat{B}_{1,2} = 0.5B_1 + 0.5B_2$ denoting the averaged state-update matrices for the overlapping modes. Important to notice is that modes 2 and 4 (which are active in sectors D_2 and D_4) are averaged due to overlapping membership functions. Using reachability analysis and computing T_j as per (17) we got

$$T_2 = \begin{bmatrix} -6.6815 & 5.3750 \\ -6.1222 & 5.0709 \end{bmatrix},\ T_4 = \begin{bmatrix} -5.3759 & 6.6815 \\ -5.0709 & 6.1222 \end{bmatrix} \tag{30}$$

The maximum approximated reachable sets $\hat{T}_j$ can be computed using (18) and are given by following axis-aligned intervals:

$$\hat{T}_2 = \begin{bmatrix} -3.7226 & 3.3634 \\ -4.7012 & 3.4356 \end{bmatrix},\ \hat{T}_4 = \begin{bmatrix} -3.3634 & 3.7226 \\ -3.4356 & 4.7012 \end{bmatrix} \tag{31}$$

The affine terms f_j in (27) and the range for the maximum allowable disturbance w_k have been computed according to (20) and (22), respectively, as

$$f_2 = \begin{pmatrix} 0.4737 \\ -0.1072 \end{pmatrix},\ f_4 = \begin{pmatrix} -0.4737 \\ 0.1072 \end{pmatrix}, \tag{32}$$

$$\begin{pmatrix} -2.4852 \\ -1.5281 \end{pmatrix} \leq w_k \leq \begin{pmatrix} 2.4852 \\ 1.5281 \end{pmatrix}. \tag{33}$$

The final PWA model of the form (8) is then composed of (27) and (30), where the regions D_j over which each dynamics is active is given by (26). The maximal range for the unknown disturbance w_k is defined by (31).

Once the PWA description is available, we can apply the minimum-time algorithm described in the previous section to design a stabilizing feedback law. The algorithm implemented in the Multi-Parametric Toolbox [14] terminated at iteration 11 and the polyhedral partition over which the control law is defined is depicted in Fig. 6. Moreover, the figure also shows closed-loop trajectories starting from various feasible initial conditions. The original fuzzy model was used as the controlled plant during the simulations.

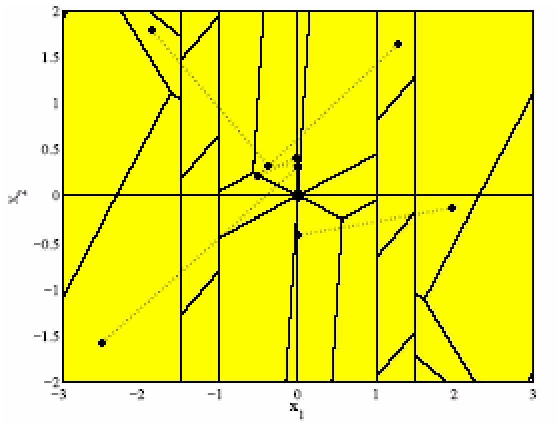

Fig. 6 Mapping of the PWA controller.

In addition, Fig. 7 shows the time evolution of the controlled state variables and the input profile for the case of $x_0 = [2.5,\ 1]^T$. As can be seen from the picture, the calculated controller drives the system states towards the origin in a dead-beat fashion.

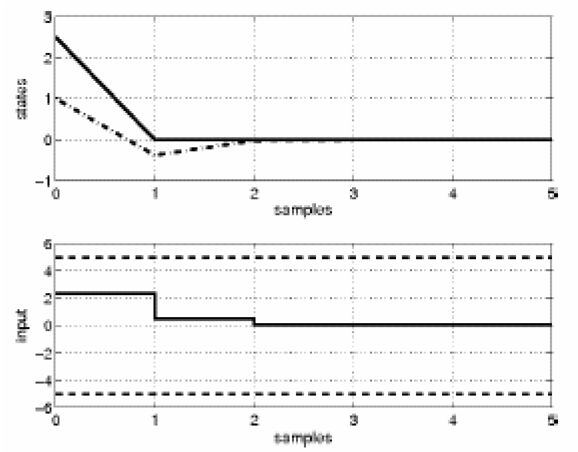

Fig. 7 Closed loop simulations.

Conclusion

This paper presents a way to incorporate input/state constraints in the control desing of the process described by Takagi-Sugeno model while preserving stability. It is shown, that the input fuzzy sets are decomposed to strictly separated region, by introducing averaging models with bounded uncertainties. Computation of uncertainties is performed via reachability analysis, which consist of a collection of polytope operations and results in a PWA model. Consequently the PWA model is adopted to design an explicit controller which guarantees closed-loop stability. The design procedure is based on a minimum-time principle in which all system states are pushed into a stabilizing terminal set in the least possible number of steps.

Acknowledgment

The authors are pleased to acknowledge the financial support of the Scientific Grant Agency of the Slovak Republic under grants No. 1/3081/06 and 1/4055/07 within the framework of the European Social Fund (PhD Students for Modern Industrial Automation in SR, JPD 3 2005/NP1-047, No 13120200115).

References

[1] VASIČKANINOVÁ, A., BAKOŠOVÁ, M.: Fuzzy modelling and identification of the chemical technological processes. In S. Krejčí, editor, *Proc. 7. Int. Scientific-Tehnical Conf. Process Control 2006*, June 13-16 2006.

[2] ALLGÖWER, F., ZHENG, A.: editors. *Nonlinear Model Predictive Control*. Birkhäuser, 2000.

[3] BEMPORAD, A., MORARI, M.: Control of systems integrating logic, dynamics, and constraints. *Automatica*, 35(3):407–427, March 1999.

[4] BORRELLI, F.: Constrained Optimal Control of Linear and Hybrid Systems. In *Lecture Notes in Control and Information Sciences*, volume 290. Springer, 2003.

[5] MENDONCA, L.F., VIEIRA, S.M., SOUSA, J.M.C.: Decision tree search methods in fuzzy modeling and classification. *International Journal of Approximate Reasoning*, 44(2):106–123, February 2007.

[6] CLARKE, D.W., MOHTADI, C., TUFFS, P.S.: Generalized predictive control – Parts I-II. *Automatica*, 23(2), 1987.

[7] CUTLER, C.R., RAMAKER, B.L.: Dynamic matrix control – a computer control algorithm. In *Proceedings, Joint American Control Conference*, San Francisco, California, USA, 1980.

[8] ESPINOSA, J.J., HADJILI, M., WERTZ, V, VANDEWALLE, J.: Predictive Control Using Fuzzy Models-Comparative Study. In *Proc. of the European Control Conference*, Karlsruhe, Germany, August, September 1999. paper f0547.pdf.

[9] GRIEDER, P., KVASNICA, M., BAOTIC, M., MORARI, M.: Stabilizing low complexity feedback control of constrained piecewise affine systems. *Automatica*, 41, issue 10:1683–1694, October 2005.

[10] HE, M., CAI, W.J., LI, S.Y.: Multiple fuzzy model-based temperature predictive control for HVAC systems. *Information Sciences*, 169(1–2):155–174, January 2005.

[11] KHABER, F., ZEHAR, K., HAMZAOUI, A.: State Feedback Controller Design via Takagi-Sugeno Fuzzy Model: LMI Approach. *International Journal of Computational Intelligence*, 2(3), 2005.

[12] KOSKO, B.: Fuzzy systems as universal approximators. In *Proceedings FUZZ'IEEE'92*, pages 1153–1162, San Diego, California, USA, 1992.

[13] KOUVARITAKIS, B., CANNON, M.:, editors. *Nonlinear predictive control: theory and practice*. IEE Control Engineering series, 2001.

[14] KVASNICA, M., GRIEDER, P., BAOTIC, M., MORARI, M.: Multi-Parametric Toolbox (MPT). In *Hybrid Systems: Computation and Control*, pages 448–462, March 2004.

[15] LI, J., WANG, H.O., BUSHNELL, L., HONG, Y.: A Fuzzy Logic Approach to Optimal Controlof Nonlinear Systems. *International Journal of Fuzzy Systems*, 2(3):153–163, 2000.

[16] LI, N., LI, S.Y., XI, Y.G.: Multimodel predictive control based on the Takagi-Sugeno fuzzy models: a case study. *Information Sciences*, 165(3–4):247–263, October 2004.

[17] MACIEJOWSKI, J.M.: *Predictive Control with Constraints*. Prentice Hall, 2002.

[18] PRETT, D.M., GARCIA C.E.: *Fundamental Process Control*. Butterworths, Boston, 1988.

[19] RAKOVIC, S.V., GRIEDER, P., KVASNICA, M., MAYNE, D.Q., MORARI, M.: Computation of Invariant Sets for Piecewise Affine Discrete Time Systems subject to Bounded Disturbances. In *Proceeding of the 43rd IEEE Conference on Decision and Control*, pages 1418–1423, Atlantis, Paradise Island, Bahamas, December 2004.

[20] SUARD, R., LÖFBERG, J., GRIEDER, P., KVASNICA M., MORARI, M.: Efficient Computation of Controller Partitions in Multi-Parametric Programming. In IEEE *Conference on Decision and Control*, Bahamas, December 2004.

[21] TAKAGI, T., SUGENO, M.: Fuzzy identications of fuzzy systems and its applications to modelling and control. *IEEE Trans. Systems Man and Cybernetics*, 15:116–132, 1985.

[22] YONEYAMA, J.: Robust stability and stabilization for uncertain Takagi-Sugeno fuzzy time-delay systems. *Fuzzy Sets and Systems*, 158(2):115–134, January 2007.

Ing. Martin Herceg

Škola: Slovenská Technická Univerzita v Bratislave
Fakulta/Ústav: Fakulta Chemickej a Potravinárskej Technológie, Ústav Automatizácie, Informatizácie a Matematiky
Katedra: Oddelenie Informatizácie a Riadenia Procesov
Ulica: Radlinského 9
PSČ a Mesto: 812 37 Bratislava
Tel.: +421 2 52495269
Fax: +421 2 52496469
E-mail: martin.herceg@stuba.sk