Application of noise filter with multivariable GPC

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Abstract
Popularity of Model Predictive Control techniques is growing in recent years. It is an intuitive and general design method convenient for controlling of multivariable plants, systems with dead-times and it enables simple constraints handling and future references. Authors deal with Generalized Predictive Control based on CARIMA process model. Control experiments with pilot plant are presented by considering the noise polynomial as a tuneable controller parameter. It is shown how the noise polynomial (data filter) improves the controller insensitivity against the high frequency uncertainties – i.e. the measurement noise.

Keywords: TITO control, generalized predictive control, CARIMA model, noise model, noise filtering

Introduction

Controlled processes in chemical industry are often multivariable systems with dead times and constraints. The control is not easy task under these conditions. Many control methods exist to deal with the individual problems and by an application they are usually combined together to get satisfactory solution. The result is that some information are implemented by the control design, some are respected passively and some are ignored. Model Predictive Control (MPC) is an open methodology how to pose control design in time domain. The idea is to optimize dynamically future process behaviour by the minimization of a selected criterion – to compute optimal future manipulated variables. Future controlled variables are computed from the process model and actual state as a response to the future manipulated variables. Quadratic finite horizon multiobjective criterion is often used – in the base form with the future control error and control movement penalization. At every sampling time all the computations are repeated with actual information (about measured variables, disturbances and future set-points) and the only first control action from the whole calculated future vector is applied – this is called as a "receding horizon strategy". It is possible to consider the future set-point, future or measurable disturbances and constraints quite easily. Necessary and key condition is ability to describe the process behaviour in some mathematical form (input-output, state-space, finite step response, finite impulse response model,…). Using of different models (and consequently different mathematical apparatus) and criterions leads to wide range of control strategies.

Generalized Predictive Control (GPC) is dealt in this paper. Controlled Auto-Regressive Integrated Moving Average (CARIMA) process model, finite horizon criterion with the future control error and control movement penalization are used. An analytical solution is possible in the case without constraints. In the general GPC case the process model contains noise model (coloured noise polynomial, polynomial matrix in MIMO case). This polynomial can arise from the process identification – as a noise characteristic. Practical effect is that this polynomial acts as a data filter and it is often used as a one of the controller tuneable parameters. This polynomial does not change the set-point tracking but decreases controller sensitivity to the measurement noise and model mismatch (e.g. by nonlinear or time variant processes). It is possible to use polynomial methods (spectral factorization) and to design optimal filter which has identical behaviour as a Kalman filter for state-space model. The aim of this paper is not the design of this polynomial but to demonstrate how the choice of different polynomials and process models influence the real control if controlled variables are burden with a measurement noise.

1. Generalized Predictive Control

Only the key assumptions and conclusions of controller design will be stated. GPC methodology and algorithms are published for example in [1], [2], [3], [4]. We consider CARIMA model for a n_y-output and n_u-input multivariable process in the form

$$\mathbf{A}(z^{-1})\cdot\mathbf{y}(k)=\mathbf{B}(z^{-1})\cdot\mathbf{u}(k-1)+\frac{\mathbf{C}(z^{-1})}{\Delta}\mathbf{e}(k) \qquad (1)$$

where $\mathbf{A}(z^{-1})$ and $\mathbf{C}(z^{-1})$ are $n_y \times n_y$ monic polynomial matrices and $\mathbf{B}(z^{-1})$ is an $n_y \times n_u$ polynomial matrix. The operator Δ is defined as $\Delta = 1 - z^{-1}$. The variables $\mathbf{y}(k)$, $\mathbf{u}(k)$, $\mathbf{e}(k)$ are $n_y \times 1$ output vector, $n_u \times 1$ input vector and $n_y \times 1$ noise vector respectively. The noise vector is supposed to be white noise with zero mean. The $\mathbf{C}(z^{-1})$ polynomial matrix which is used in the prediction model is denoted by $\mathbf{T}(z^{-1})$ further on and it is called robustness filter in GPC terminology [1].

Following finite horizon quadratic criterion is considered

$$J(N_1,N_2,N_u)=\sum_{j=N_1}^{N_2}\left\|\hat{\mathbf{y}}(k+j\mid k)-\mathbf{y}_r(k+j)\right\|_{\mathbf{R}}^2+ +\sum_{j=1}^{N_u}\left\|\Delta\mathbf{u}(k+j-1)\right\|_{\mathbf{Q}}^2 \qquad (2)$$

where $\hat{\mathbf{y}}(k+j\mid k)$ is an optimum j-step ahead prediction of the system output, N_1 and N_2 are the minimum and maximum prediction horizon and $\mathbf{y}_r(k+j)$ is a future set-point for the controlled variables. N_u is the control horizon (after the first N_u control moves the control signal is kept constant). **R** and **Q** are positive definite weighting matrixes.

The optimum *j*-step ahead prediction of the system output can be written in matrix form as

$$\mathbf{Y}_{N12} = \mathbf{G}_{N12u} \cdot \Delta\mathbf{U}_{Nu} + \mathbf{f}_{N12} = \mathbf{G}_{N12u} \cdot \Delta\mathbf{U}_{Nu} + \\ + \mathbf{G}'_{N12} \cdot \mathbf{U}_{\mathrm{p}} + \mathbf{F}_{N12} \cdot \mathbf{Y}_{\mathrm{p}} \qquad (3)$$

where the first term is so called forced and the second term free response of the system. The vector $\Delta\mathbf{U}_{Nu}$ is vector of future control movements to be calculated, the matrices $\mathbf{G}_{N12u}$, $\mathbf{G'}_{N12}$ and $\mathbf{F}_{N12}$ are evaluated from the process model and vectors $\mathbf{U}_{\mathrm{p}}$ and $\mathbf{Y}_{\mathrm{p}}$ are vectors of past process inputs and outputs

$$\mathbf{U}_{\mathrm{p}} = \left[\Delta\mathbf{u}(k-1)^T \ldots \Delta\mathbf{u}(k-n_b)^T\right]^T, \quad n_b = \delta\left(\mathbf{B}\left(z^{-1}\right)\right) \\ \mathbf{Y}_{\mathrm{p}} = \left[\mathbf{y}(k)^T \ldots \mathbf{y}(k-n_a)^T\right]^T, \quad n_a = \delta\left(\mathbf{A}\left(z^{-1}\right)\right)+1 \qquad (4)$$

Criterion (2) in matrix form is

$$J = \left(\mathbf{G}_{N12u} \cdot \Delta\mathbf{U}_{Nu} + \mathbf{f}_{N12} - \mathbf{Y}_{r,N12}\right)^T \cdot \overline{\mathbf{R}} \cdot \left(\mathbf{G}_{N12u} \cdot \Delta\mathbf{U}_{Nu} + \mathbf{f}_{N12} - \mathbf{Y}_{r,N12}\right) + \\ + \Delta\mathbf{U}_{Nu}{}^T \cdot \overline{\mathbf{Q}} \cdot \Delta\mathbf{U}_{Nu} \qquad (5)$$

where $\overline{\mathbf{R}} = diag(\mathbf{R}, \ldots, \mathbf{R})$, $\overline{\mathbf{Q}} = diag(\mathbf{Q}, \ldots, \mathbf{Q})$.

If there are no constraints the optimum can be expressed as

$$\Delta\mathbf{U}_{Nu} = \left(\mathbf{G}_{N12u}{}^T \cdot \overline{\mathbf{R}} \cdot \mathbf{G}_{N12u} + \overline{\mathbf{Q}}\right)^{-1} \cdot \mathbf{G}_{N12u}{}^T \cdot \overline{\mathbf{R}} \cdot \left(\mathbf{Y}_{r,N12} - \mathbf{f}_{N12}\right) = \\ \mathbf{L} \cdot \left(\mathbf{Y}_{r,N12} - \mathbf{f}_{N12}\right) \qquad (6)$$

Receding strategy means that only the first element of the sequence $\Delta\mathbf{U}_{Nu}$ is actually sent to the process and the control action is then

$$\Delta\mathbf{u}(k) = \mathbf{K}\left(\mathbf{Y}_{r,N12} - \mathbf{f}_{N12}\right) \qquad (7)$$

where $\mathbf{K}$ are the first n_u rows of matrix $\mathbf{L}$ and $\mathbf{Y}_{r,N2}$ are the future set-points

$$\mathbf{Y}_{r,N12} = \left[\mathbf{y}_r(k+N_1)^T \ldots \mathbf{y}_r(k+N_2)^T\right]^T.$$

2. Controlled process

Control experiments with laboratory hydraulic-pneumatic system (HPS - see Fig 1) are presented to demonstrate the effect of polynomial matrix $\mathbf{T}(z^{-1})$ in "real conditions" – in the case of measurement noise, disturbances, model mismatch and nonlinearity.

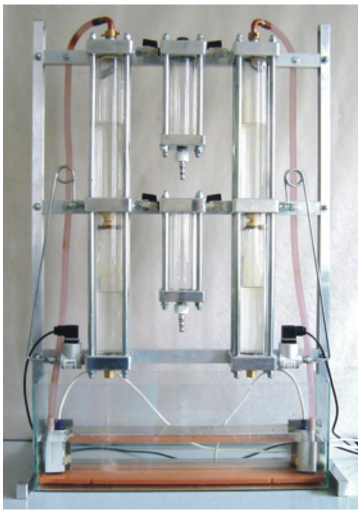

Fig.1 Hydraulic-pneumatic system

Laboratory model HPS is described more in detail e.g. in [6]. Water is pumped to the high tanks and flows out though orifices in the tanks bottom to the low tanks and then to the reservoir. Pneumatic volumes above the water levels are closed from atmosphere and connected between two adjacent tanks. Air tanks are connected to the air pipes for increasing the capacity - dynamics. Orifices serving as a limited connection with the atmosphere are in the air tanks bottom. That way an atypical dynamical system behavior is obtained. Change in one pump flow rate originates water level change in the same section and also air pressure change. This pressure influences water level in second section too. Pressure in air volume is gradually equalized with the atmospheric pressure (is going out to the atmosphere). Then the cross coupling has character as a dynamical derivative term with zero steady-state gain.

Levels in low water tanks are measured indirectly by the difference pressure sensors. Output signals from the pressure sensors y_L, y_R are electric voltage signals in range from 0 to 10 V. The water flow rates are controlled by supply voltage of the pumps. The process input signals u_L, u_R are voltage signals in range from 0 to 10 V which are transformed to the range from 4 to 10 V and gained in pump control unit.

3. Process model

Nonlinear model of HPS is obtained by an application of mathematical-physical analysis – physical laws by respecting the system construction [5]. The high air tank is opened to the atmosphere by all experiments and the orifice is placed only in the low air tank. This configuration has advantage that the pump static characteristics are only functions of the pump supply voltage. Interesting feature of this configuration for the control is that the levels in high water tanks tend to overflow or flow out.

Nonlinear model is analytically linearized – state-space and input-output linear process models are obtained. Unknown static parameters of pumps, water tanks and pressure sensors models are estimated by a numerical optimization method from the measured static characteristics. It is not possible to estimate the air discharge coefficient for the orifice in the low air tank from static data. Dynamical responses of the water levels in low water tanks are measured and the air discharge coefficient is estimated by the use of nonlinear model and numerical optimization method.

Parameters of input-output model are evaluated in the middle of the HPS working region. The linearization point is in the Table 1 - the outputs are calculated from the nonlinear model for given inputs.

Position	u [V]	y [V]
Left - L	5,0	5,6
Right - R	5,0	5,7

Tab.1 Working point

The input-output model of HPS (in the following text called as a "full model") is

$$\begin{bmatrix}\Delta Y_{LL}\\ \Delta Y_{RL}\end{bmatrix}=\frac{1}{A}\begin{bmatrix}B_{LL} & B_{LR}\\ B_{RL} & B_{RR}\end{bmatrix}\cdot\begin{bmatrix}\Delta U_{L}\\ \Delta U_{R}\end{bmatrix}$$

$$B_{LL}=316{,}8\,p^3+523\,p^2+15{,}89\,p+0{,}1486$$

$$B_{LR}=-616{,}8\,p^2-7{,}83\,p$$

$$B_{RL}=-619{,}1\,p^2-12{,}02\,p \qquad (8)$$

$$B_{RR}=740{,}2\,p^3+1223\,p^2+24{,}29\,p+0{,}1484$$

$$A=3{,}593.10^5\,p^5+1{,}075.10^6\,p^4+9{,}026.10^4\,p^3+ +2622\,p^2+29{,}05\,p+0{,}1086$$

Rather complicated nonlinear model is obtained by the mathematical-physical analysis. Also model (8) has quite high order (corresponding to the number of process capacities).

Therefore another simpler model is identified from the measured step responses (Fig. 2 – the measured step responses are plotted with the solid line, the simulated with the dotted one) – this model is in the following text called as a "simplified model".

$$\begin{bmatrix}\Delta Y_{LL}\\ \Delta Y_{RL}\end{bmatrix}=\frac{1}{A}\begin{bmatrix}1{,}484\cdot(33{,}37\,p+1) & -66{,}1\,p\\ -99{,}11\,p & 1{,}540\cdot(94{,}79\,p+1)\end{bmatrix}\cdot\begin{bmatrix}\Delta U_{L}\\ \Delta U_{R}\end{bmatrix} \qquad (9)$$

$$A=(96{,}84\,p+1)(99{,}11\,p+1)$$

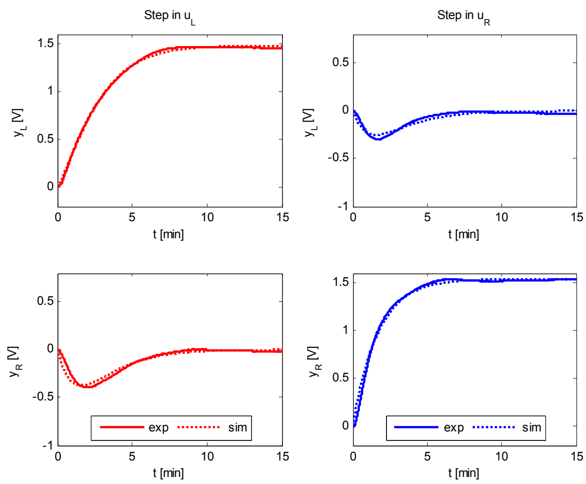

Fig.2 Step responses – simplified model

4. Control experiments

Following control experiment is considered: experiment starts with the set-points voltages for both lower tanks water levels 4 V, in time 10 minutes the set-point for the left water level is stepwise changed to 6 V and kept constant till the end of the experiment and identical set-point change is realized for the right water level in time 20 minutes (see e.g. Fig. 5).

GPC parameters are listed in the Table 2. Process model with noise polynomial matrix $\mathbf{T}(z^{-1})$ is used by the predictive controller (Coloured noise GPC – polynomial matrix $\mathbf{T}(z^{-1})$ is considered as a design parameter – as a filter). Three polynomial matrices $\mathbf{T}(z^{-1})$ are tested – diagonal polynomial matrices with diagonal elements $T = 1$ (special case - white noise GPC), $T = 1\text{-}0.8z^{-1}$ and $T = (1\text{-}0.8z^{-1})^2$. Controllers with two process models are studied – with model arisen from the linearization of mathematical-physical nonlinear model – full model (8) and with a model estimated from the step responses - simplified model (9).

T_S	10 s	
	Left section	Right section
N_1	1	1
$N_2 = N_u$	18	18
R	$\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$	$\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$
Q	$\begin{bmatrix}5 & 0\\ 0 & 5\end{bmatrix}$	$\begin{bmatrix}5 & 0\\ 0 & 5\end{bmatrix}$

Tab.2 GPC parameters

Two criteria of control quality are evaluated by all experiments – sum of absolute control movements for manipulated variable K_u and absolute control error area for controlled variable K_y (10). Both values are referred to the experiment time duration ($N.T_S$). Criteria are analyzed for both set-point changes separately as for two control experiments and displayed in the figures.

$$K_u=\frac{\sum_{i=1}^{N}|\Delta u(i)|}{N\cdot T_S},\quad K_y=\frac{\sum_{i=1}^{N}|y_r(i)-y(i)|}{N} \qquad (10)$$

Steady-state process output is measured (Fig. 3) - output of the pressure sensors y_L and y_R, input signal was kept constant $u_L = u_R = 5$ V and standard deviation of the signal is evaluated (Table 3).

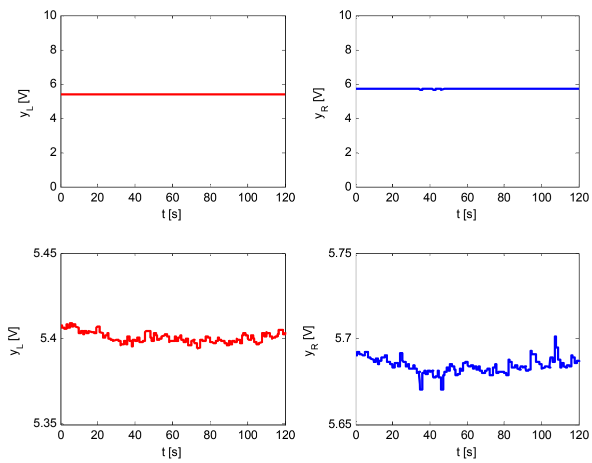

Fig.3 Measured steady-state process output

	Output signal y_L	Output signal y_R
σ	0.0032 V	0.0045 V

Tab.3 Noise "and disturbance" standard deviation

Nonlinear model [5] is controlled in all simulated control experiments. To emulate the real process behaviour additive noises *n* are added to the model outputs (pseudo-random numbers from range $\pm 0.0125\,\text{V}$) and load disturbances *d* are simulated (noises with random period 0-200 s and amplitudes - pseudo-random numbers from range $\pm 0.1\,\text{V}$).

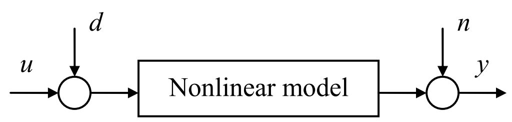

Fig.4 Nonlinear model with noise and disturbance

4.1 GPC based on full model

Simulated control experiment without noise and disturbance for polynomial *T* = 1 (white noise GPC) is shown on Fig. 5. Simulated control experiments with noise and disturbance and polynomials *T* = 1, *T* = 1-0,8z^{-1} and *T* = (1-0,8z^{-1})^2 respectively are plotted on Fig. 6, 7 and 8. Real control experiment for polynomial *T* = (1-0,8z^{-1})^2 is shown on Fig. 9.

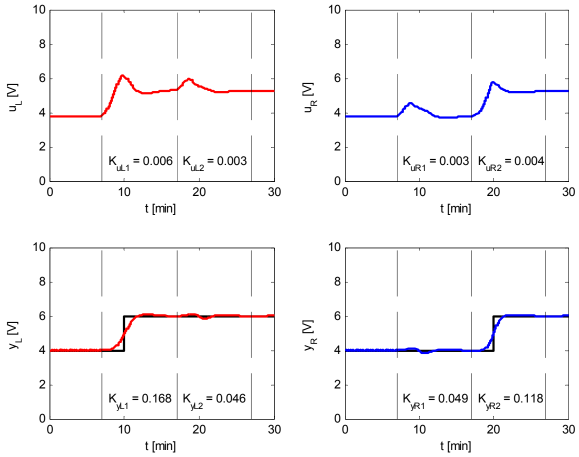

Fig.5 Simulated control without noise – full prediction model, *T* = 1

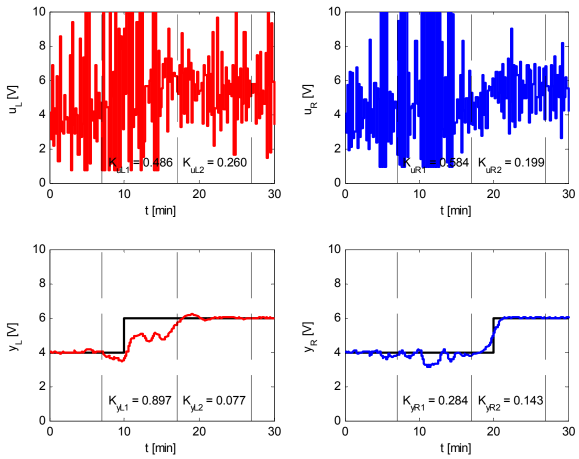

Fig.6 Simulated control with noise – full prediction model, *T* = 1

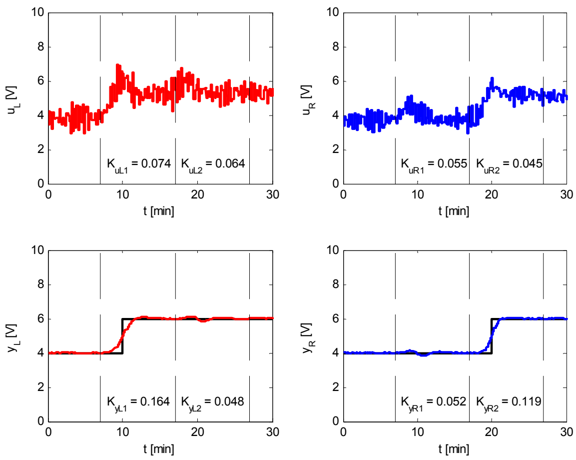

Fig.7 Simulated control with noise – full prediction model, *T* = 1-0,8z^{-1}

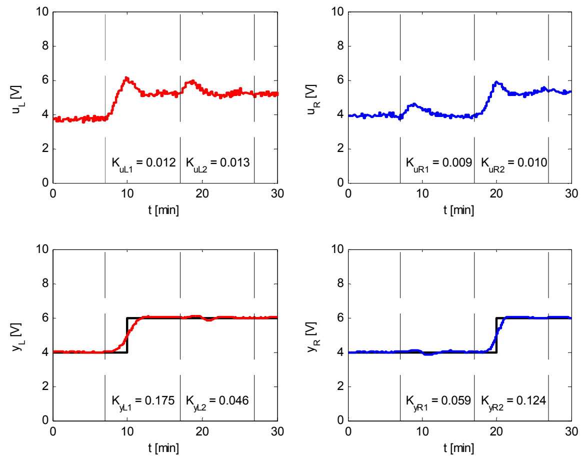

Fig.8 Simulated control with noise – full prediction model, *T* = (1-0,8z^{-1})^2

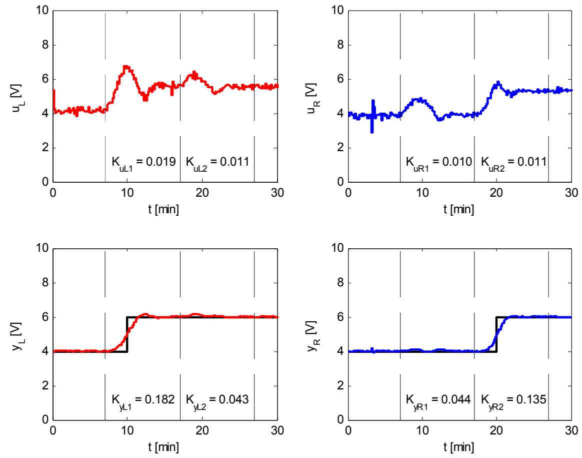

Fig.9 Real control – full prediction model, *T* = (1-0,8z^{-1})^2

4.2 GPC based on simplified model

Simulated control experiment without noise and disturbance and for polynomial *T* = 1 (white noise GPC) is shown on Fig. 10. Simulated control experiments with noise and disturbance and polynomials *T* = 1, *T* = 1-0,8z^{-1} and *T* = (1-0,8z^{-1})^2 respectively are plotted on Fig. 11, 12 and 13. Real control experiment for polynomial *T* = 1 is shown on Fig. 14. The real control provides relatively good control performance even without polynomial filter (the controller is less sensitive to the measurement noise than the one based on the full model).

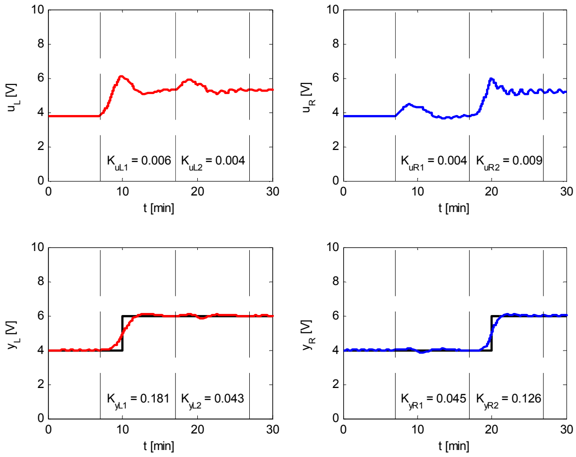

Fig.10 Simulated control without noise – simplified prediction model, *T* = 1

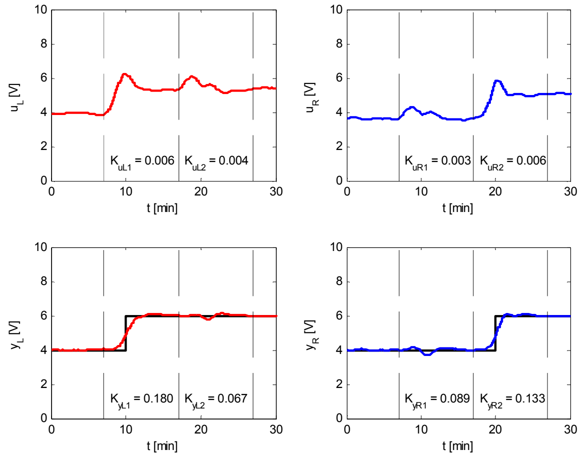

Fig.13 Simulated control with noise – simplified prediction model, $T = (1\text{-}0{,}8z^{-1})^2$

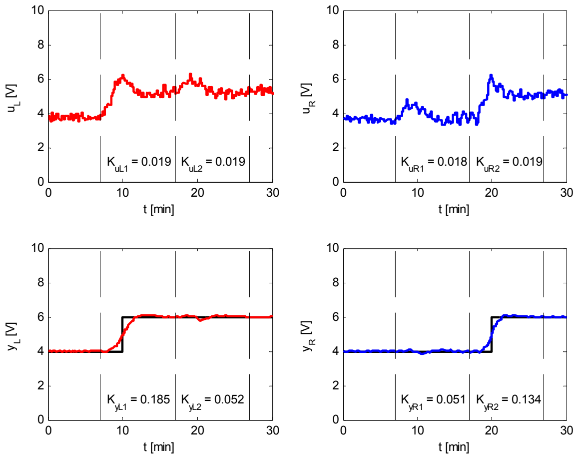

Fig.11 Simulated control with noise – simplified prediction model, *T* = 1

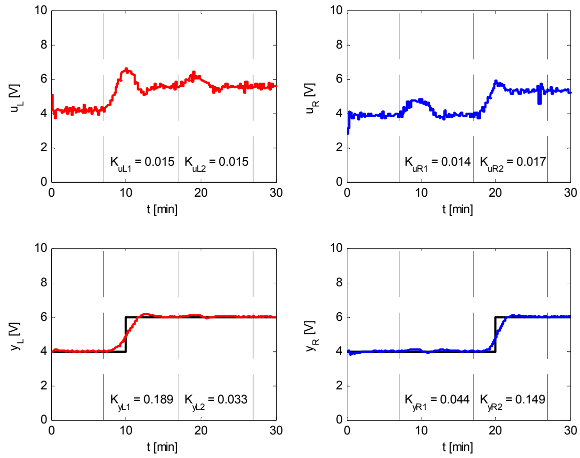

Fig.14 Real control – simplified prediction model, *T* = 1

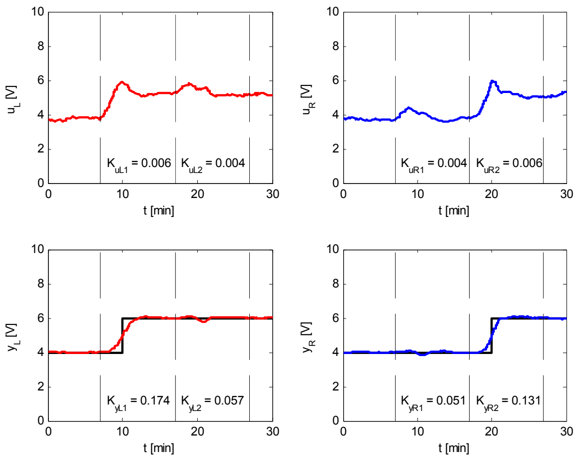

Fig.12 Simulated control with noise – simplified prediction model, $T = 1\text{-}0{,}8z^{-1}$

Conclusion

Control technique "Coloured noise Generalized Predictive Control" is treated in the paper – process model includes noise model - polynomial matrix **T**(z^{-1}) differs from **I**. Simulated and real experiments with hydraulic-pneumatic system (laboratory plant) are shown to demonstrate the effect of filter polynomial in the case of measurement noise presence. Two different models are used by GPC controller – "full" arising from linearization of nonlinear mathematical-physical model and "simplified" as an approximation of measured step responses. Control with full model is much more sensitive to the measurement noise than with the simplified model in case of white noise GPC (case without noise model, $T = 1$) – compare Fig. 6 and 11. The disadvantage of simplified model is that even in the case without noise (simulated experiment) the controlled variable is slightly oscillating – compare Fig. 5 and 10. This may be caused by the model mismatch or nonlinearity of the process. The effect of filter polynomial is clearly seen from Fig. 6, 7 and 8 – the manipulated variable is much smoother for higher filter order. The disadvantage of using the filter would be slowing down the disturbance rejection – in our case there is no provable relation with the criterion K_y.

Real experiments give very similar results for full model with $T = (1\text{-}0{,}8z^{-1})^2$ and simplified model with $T = 1$ (see Fig. 9 and 14). Advantage of the mathematical-physical model is that we dispose with the nonlinear state-space model, the states have physical meaning and we can evaluate the linear model in different working points. Next profit of this model type is that it opens nature way for nonlinear control, state-constrained control or different types of optimisations. The disadvantage is rather time and information consuming approach to get this model and to design the controller. On the other hand model from experimental identification is simple and fast to obtain and in our case we can fulfil the control demands similar as with the full model and the controller is less sensitive to the measurement noise.

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