Constrained control of the plant with the slow and fast mode

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Abstrakt
The paper considers constrained pole assignment control of the plant with two different modes (one slow and one fast) that represents one of the typical benchmark examples considered in the PID control design. For the controllers based on the double first order approximations (that correspond to the 2^{nd} order system with one zero), simple control algorithms respecting the given control constraints are presented.

Key words: constrained control, pole assignment control, PID control

Introduction

Let us consider control of the 2nd order system consisting of two parallel first order plants: with a fast and with a slow mode described by the transfer function

$$F(s)=\frac{k_1}{1+T_1 s}+\frac{k_2}{1+T_2 s}=$$
$$=K\frac{1+T_0 s}{(1+T_1 s)(1+T_2 s)}; \qquad (1)$$
$$K=k_1+k_2\ ;\ T_0=\frac{k_1 T_2+k_2 T_1}{(k_1+k_2)}$$

Such a plant seems to be quite frequent in practice – it was e.g. introduced in the „Benchmark systems for PID control" proposed by Åström and Hägglund [2], or explored in the work by Åström et al. [1]. Such systems can be met in controlling thermal plants, when the heat is transferred (see e.g. the article on the heat transfer in Wikipedia http://en.wikipedia.org/wiki/Heat_transfer) by:

- conduction,
- radiation and
- convection

We came into contact with controlling such plant when developing scaled thermal plant for the educational use (Fig.1).

By neglecting convection (this possibility of the heat transfer seems to have just a marginal effect in our case), the system can be approximated by two differently fast modes. The fast channel corresponds to a heat transfer by radiation, while the slow mode to conduction through the plant skeleton.

The first problem is related to the plant identification: as it is mentioned by Åström and Hägglund [2] „simple tuning rules usually based on the step responses normally not give good tuning for systems of this type because it is difficult to get a good estimate of the gain and the time constant."

In the work by Åström et al. [1] you can find even more: "the static behaviour is dominated by the slow mode", or "the step response is dominated by the slow time constant, but it is the faster modes that are critical for the closed loop systems" and "most attempts to tune the system based on step response data will give poor results". Their results derived by the non-convex optimization show overshooting for both considered maximum sensitivity values $M_s=1.4$ and $M_s=2.0$. Is it not possible to improve the control quality by relatively simple means?

Fig.1 The thermo-optical plant

Plant identification

Finally, the plant identification shown to be not so serious problem as mentioned by Åström and Hägglund [2] and we were able to develop a relatively simple method appropriate for the controller tuning.

For a reliable controller tuning, the measured step response (Fig.2) was approximated by the two modes and dead time as

$$S(s)=\left[\frac{k_1}{T_1 s+1}+\frac{k_2}{T_2 s+1}\right]e^{-T_d s}=$$

$$= K \frac{1+T_0 s}{(1+T_1 s)(1+T_2 s)} e^{-T_d s} \; ;$$
$$K = k_1 + k_2 \; ; \; T_0 = \frac{k_1 T_2 + k_2 T_1}{(k_1 + k_2)} \qquad (2)$$

whereby:
k_1=1.033; k_2=5.27; T_1=184.4; T_2=931.7; T_d=10;

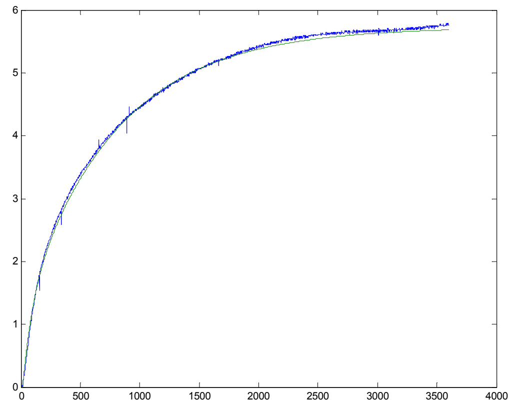

Fig.2 The measured plant step response and its approximation (2)

In the literature, the model (2) is also denoted as SOSPD model (Second Order System Plus Deadtime) with a negative zero. O'Dwyer [6] gives PID tuning rules for such a plant and 5 basic options: the ideal PID controller, the controller with filtered derivative, the classical (cascade, series, interacting) controller and the non-interacting controller with one or two degrees of freedom. In this paper, we will briefly explore simple first order plant approximations and the associated PI_1 controller and then develop the (optimal) constrained pole assignment solution and its extension by reconstruction and compensation of possible piecewise constant disturbances.

PI_1 controller

The first attempts to control the thermal plant were based on using PI_1 controller (Fig.3) that can be built up by extending the P-controller by the disturbance reconstruction and compensation based on the inverse plant model based [4], [5]. The transfer function $S(s)$ describes the dominant first-order dynamics. Because it is not so simply to say, what it means "the dominant first order dynamics", several approximations were proposed.

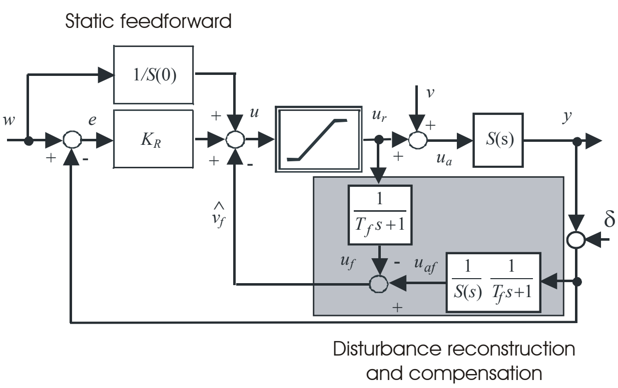

Fig.3 PI_1 controller: P-controller extended by the disturbance reconstruction and compensation

Controller based on single integrator approximation

In this case, (inspired by Ziegler and Nichols) the plant step response is approximated by the first order integrator + dead time. It means that

$$S(s) = \frac{K_s}{s} \; ; \; \frac{1}{S(0)} = 0 \qquad (3)$$

For the sake of simplicity, we set

$$K_s = \frac{k_1}{T_1} \qquad (4)$$

The identified dead time is used for choosing the fastest possible closed loop pole that is simultaneously defining the reconstruction filter time constant T_f and the P controller gain K_R

$$\alpha = \frac{1}{eT_d} \; ; \; T_f = -\frac{1}{\alpha} \; ; \; K_R = -\frac{\alpha}{K_s} \qquad (5)$$

The transient responses achieved by simulation for the plant model defined by (3-4) are in Fig.4. The response achieved by simulation corresponds to the measured responses that are typical by the control signal oscillations around the zero dynamics. The output response is typically overdamped. The results seem to be usable but they are far from those expected – the control transient shows too much oscillations.

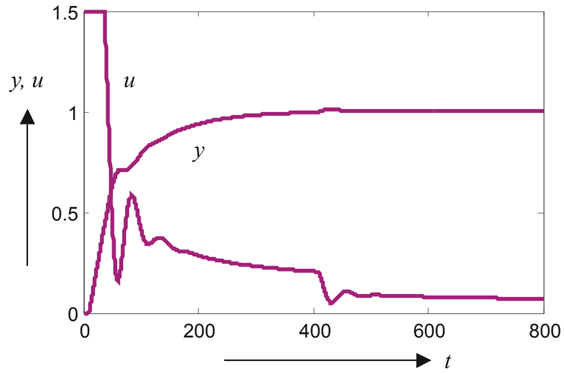

Fig.4 Control of the plant (2) by the PI_1 controller based on the integral step response approximation (3-4). In the middle of the transient an input step disturbance was applied

Controller based on single time constant approximation

Because the previous approach did not satisfy the original expectation, we were looking for reasons of the unsatisfactory behaviour. The first idea was to verify, if the problem is not caused by the integral approximation of the plant with evidently static behaviour. Therefore, the dominant dynamics was modified to

$$S(s) = \frac{k_1}{T_1 s + 1} = \frac{K_s}{s+a} \; ;$$
$$K_s = \frac{k_1}{T_1} \; ; \; \frac{1}{S(0)} = \frac{1}{k_1} \qquad (6)$$

The closed loop pole and the controller gain were specified according to (5).

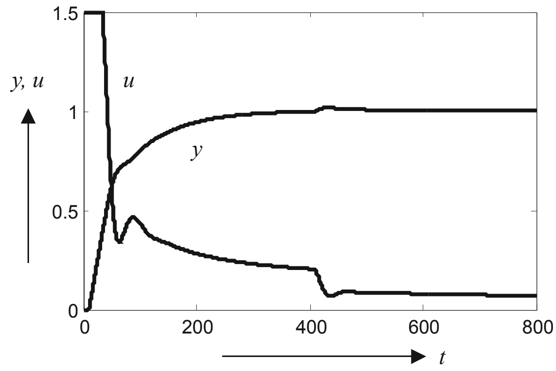

Fig.5 Control of the plant (2) by the PI$_1$ controller based on the plant approximation (6)

The corresponding simulation results are in Fig.5. Since these approximation results are still not close to the expected ones, we have checked also the first order approximation

$$S(s)=\frac{k_1+k_2}{T_1 s+1}\ ;\ \frac{1}{S(0)}=\frac{1}{k_1+k_2}\ ;\qquad K_s=\frac{k_1+k_2}{T_1} \tag{7}$$

As a result, the control oscillation around the zero dynamics disappeared, but an overshooting appeared (Fig.6).

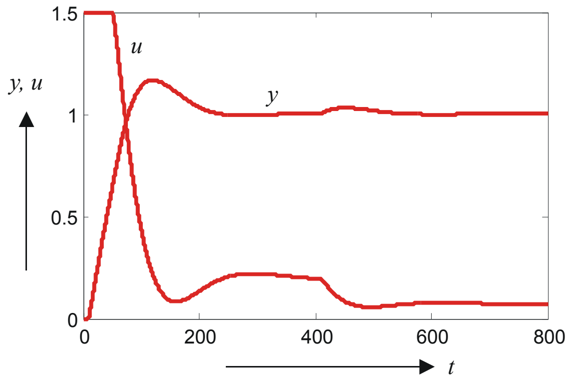

Fig.6 Control of the plant (2) by the PI$_1$ controller based on the plant approximation (7)

P-P controller

This notion was inspired by works of Åström et al. [1] that tried to develop general parameterized solutions which can be relatively easily adjusted to a particular situation by building on parameterizations as the sensitivity functions, or the complementary sensitivity functions that are related to the robust control. Having clear-cut physical interpretation of the effect of such tuning parameters and clear picture of its appropriate default values, the tuning should be much simpler and reliable.

However, from the point of view of the constrained control the sensitivity and complementary sensitivity functions do not represent an optimal solution. They e.g. do not mach the natural expectation that by decreasing the range of possible parameter fluctuations, the effect of the non-modelled dynamics (parasitic delays) and the amplitude of the measurement noise - when there are no other specifications on the control quality - the achieved solutions would converge to the results of the minimum time control.

Such a requirement was obviously followed using another way of the closed loop parameterization – the pole assignment method by Glattfelder and Schaufelberger [7]. The anti-windup PI controller they have analyzed was very close to the ideal control signal step reaction converging to the one pulse of the minimum time control. But not completely.

Because the simplified solutions based on the first order approximation did not yield satisfactory control quality, the next step was to explore the 2nd order plant approximations with the relative degree 1.

Considering two parallel first order channels, the output variable of the particular channels may be described by the differential equations

$$\dot y_1=\frac{k_1 u-y_1}{T_1}\ ;\ \dot y_2=\frac{k_2 u-y_2}{T_2} \tag{8}$$

For the output variable it holds

$$y=y_1+y_2 \tag{9}$$

Therefore, it also holds

$$\dot y=\dot y_1+\dot y_2=\bar K u-\frac{1}{T_1 y}-\tau\, y_2;\qquad \bar K=\left(\frac{k_1}{T_1}+\frac{k_2}{T_2}\right);\ \tau=\left(\frac{1}{T_2}-\frac{1}{T_1}\right) \tag{10}$$

For a piecewise constant setpoint signal, the control error is defined as

$$e=w-y\ ;\ \dot e=-\dot y \tag{11}$$

The control signal u_w required for maintaining the reference value $w=const$ can be derived as

$$u_w=\frac{w}{k_1+k_2} \tag{12}$$

In the steady states, the outputs of the particular channels will be given as

$$y_{1\infty}=K_1 u_w=\frac{k_1}{k_1+k_2}w\ ;\qquad y_{2\infty}=k_2 u_w=\frac{k_2}{k_1+k_2}w\overset{!}{=}w_2 \tag{13}$$

The pole assignment control of the first order systems is described by the differential equation

$$\dot e=\alpha e \tag{14}$$

whereby α is the chosen closed loop pole. After substituting for the control error into the above equation, one gets

$$-\bar K u+\frac{y-w+w}{T_1}+\tau\left(y_2-w_2+w_2\right)==\alpha\left(w-y\right) \tag{15}$$

A short manipulation gives the paralel P-P controller

$$u=\frac{1}{K}w+K_R e+K_{R2}e_2\ ;\qquad e_2=w_2-y_2\ ;\ K_R=-\frac{\alpha+1/T_1}{\bar K}\ ;\qquad K_{R2}=-\frac{T_1-T_2}{k_1T_2-k_2T_1} \tag{16}$$

For the relatively slow closed loop poles the transient response is similar to controlling simple first order system

(Fig.7) for the relatively fast dynamics (which is close to the minimum time control) the influence of the zero dynamics is already evident: after the output reaches the reference value, the control signal is not constant, but it slowly varies with the zero dynamics determined by the time constant T_0.

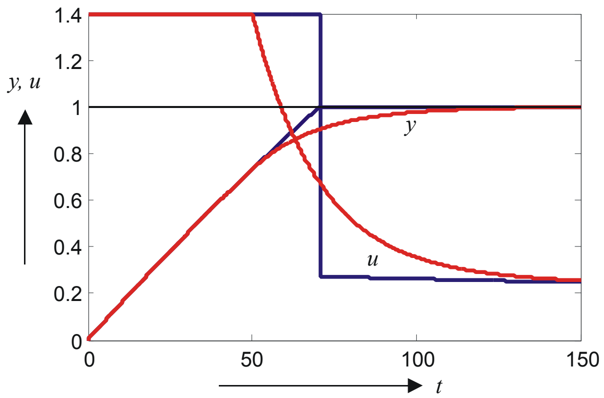

Fig.7 Transient responses for $\alpha_1 = -0.05$ (red) and $\alpha_2 = -20$ (blue) k_1=1.0; k_2=5.3; T_1=184; T_2=931; T_d=0

Reconstruction of the auxiliary output y_2

Since in practical applications it will mostly be considered just measuring of the output y, the controller realization will require reconstruction of the auxiliary output y_2.

The relation between the control and the output signal can be characterized by the transfer function (1). The output y_2 can be reconstructed either from the known control signal as

$$Y_2(s) = \frac{k_2}{1+T_2 s} U(s) \tag{17}$$

or from the measured output as

$$Y_2(s) = \frac{k_2}{K} \frac{1+T_1 s}{1+T_0 s} Y(s) \tag{18}$$

The advantage of the first possibility may be lower noise influence, the advantage of the second possibility higher robustness against acting disturbances.

PI_1-P controller

The thermal plant is typically a system with slowly varying parameters, so that the use of the P-P control structure would cause possible existence of the permanent control error. Therefore, it is mostly appropriate to extend the controller by the disturbance reconstruction and compensation via inverse model. Since we have already reconstructed the output y_2, next the input disturbance can be simply reconstructed in the same way as it was done in the structures based on the first order plant approximation (Fig.3) according to

$$V_r(s) = \frac{1}{k_2} \frac{1+T_2 s}{1+T_f s} Y_2(s) - \frac{1}{1+T_f s} U(s) \tag{19}$$

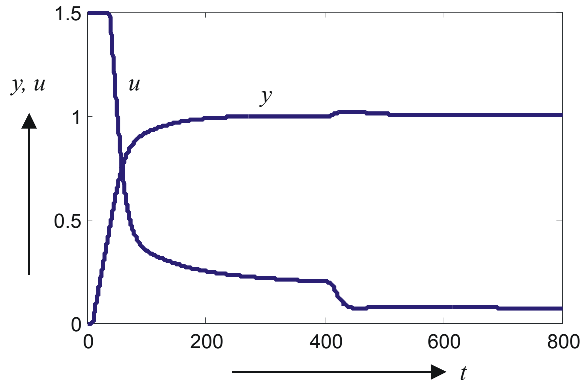

Fig.8 Control of the plant (2) by the PI_1-P controller according to Fig.9

For the controller tuning (4-5) the achieved transient responses (Fig.8) show that the control signal response is already close to one interval of the minimum time control. The control signal transient from the limit value to the steady state may be tuned by the closed loop poles sufficiently soft to satisfy also majority of industrial applications and not to excite the always hidden high-frequency modes corresponding to parasitic time delays.

Conclusions

The newly developed solution shows possibility to control the thermal plant and other systems with the dominant 2nd order dynamics and with the relative degree 1 by means of the paralel P-P, or PI_1-P control structures that give dynamics scalable by the closed loop pole and ranging from the fully linear one up to the minimum time control.

Due to the relative degree lower than the plant degree, the minimum time control, however, differs from the usual bang-bang control. Besides of one control interval a saturation limit it includes also one interval of control determined by the zero dynamics. So, it can be classified as a special case of the singular optimal control.

This approach yielding transients from the dynamical class 1 (with one possible control interval at the saturation limit) can also be generalized for the plant approximations with higher number of parallel first order channels. In the particular case of the thermal systems this, however, seems to be not necessary, since the transients achieved by the real time control are in good conformity with the expected (simulation) results.

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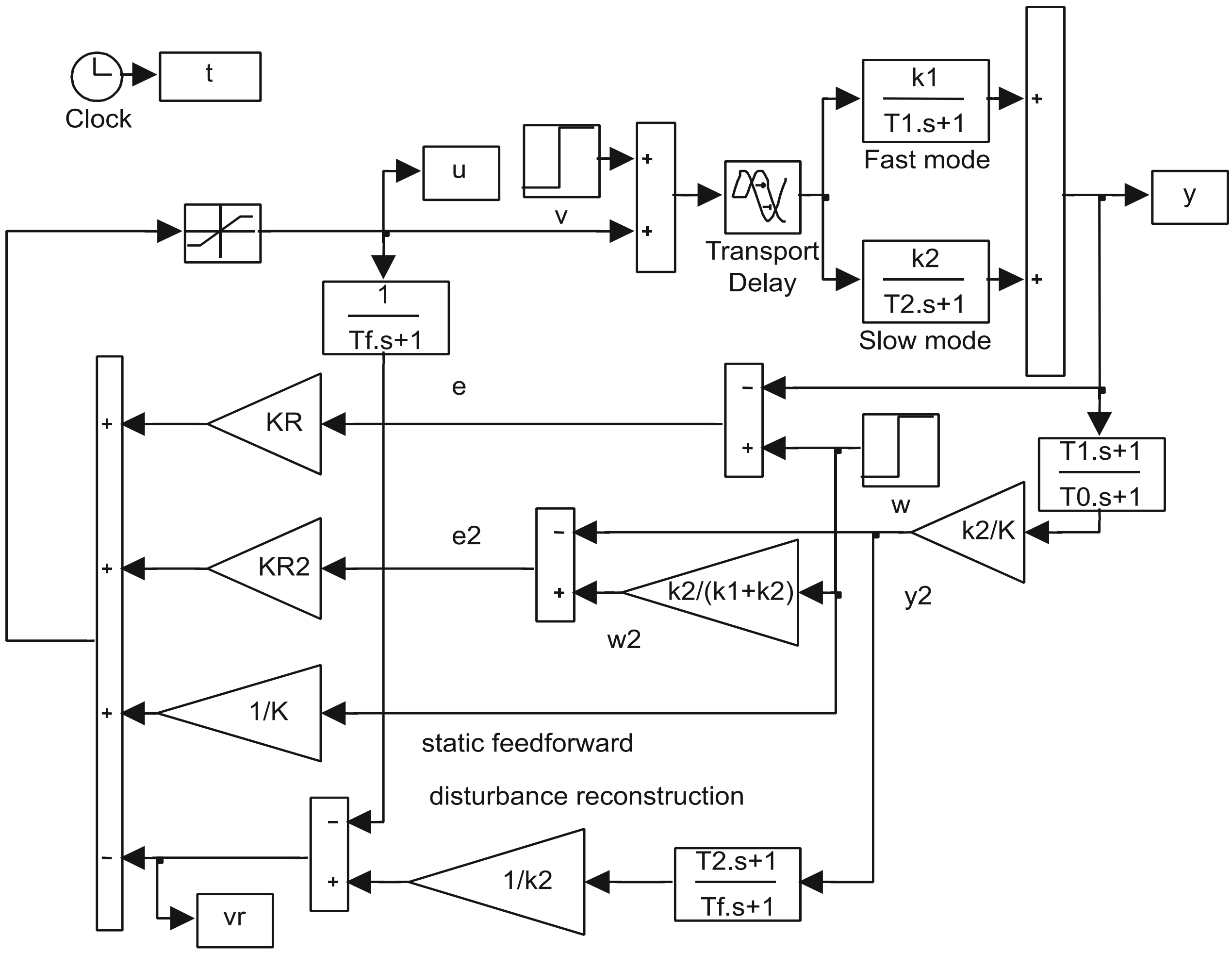

Fig.9 Simulation scheme of the PI$_1$-P controller