Residual generators design using eigenstructure assignment

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Abstract
Some aspects of the model-based robust residual generation with discrete-time state observers, as well as the eigenstructure based design procedures are presented in the paper. The design task is to modify the observer state matrix in a prescribed manner where the extra freedom is used to decouple the disturbance and some part of outputs. An extended exposition of the problem is presented here to handle the special case of the pole placement eigenvalue set, and an example is given to illustrate the algorithm performance.

Keywords: Fault diagnosis, residual generation, state observer, eigenstructure assignment, disturbance decoupling

Introduction

Automated diagnosis has been one of the more fruitful applications in sophisticated control systems, with potential significance for domains in which diagnosis of systems must proceed while the system is operative and testing opportunities are limited by operational considerations. The real problem is usually to fix the system with faults so that system can continue its mission for some time with some limitations of functionality. Thus, diagnosis is only part of larger problem known as fault detection, identification and reconfiguration (FDIR). The practical benefits of an integrated approach to FDIR seem to be considerable, especially when knowledge of available fault isolations and system reconfigurations is used to reduce the cost and increase the reliability and utility of control and diagnosis.

Therefore, the essential aspect for the design of fault-tolerant control requires the conception of diagnosis procedure that can solve the fault detection and isolation problem. This procedure composes residual generation (that contain information about the failures or defects) followed by their evaluation within decision functions. To failures or process disturbance detection, analytical redundancy is used to generate residuals that are based on implicit information in functional or analytical relationships, which exist between measurements taken from the process. A fault in the fault diagnosis systems can be detected and located when has to cause a residual change and subsequent analyze of residuals have to provide information about faulty component localization.

In this note one type of robust residual generation is considered. Starting with observer based residuals, the solution is carried out along the standard eigenstructure assignment, and using the optimal strategy for disturbance decoupling the problem of partly output decoupling is outlined, to calculate the estimator gain matrix. An interesting point is that presented design has the essential structure of a standard disturbance decoupling algorithms.

Used principle can be viewed as extension to the method applied in [1]. An example is presented to demonstrate the role of eigenstructure assignment in the optimization procedure, where the solution is easily achieved by solving for a set of desired observer eigenvalues.

1. Observer based residuals

Generally, a discrete-time linear dynamic system with sensor and actuator faults can be modeled by equations

$$\mathbf{q}(i+1) = \mathbf{F}\mathbf{q}(i) + \mathbf{G}(\mathbf{u}(i) + \mathbf{f}_a(i)) + \mathbf{E}\mathbf{d}(i) \qquad (1)$$

$$\mathbf{y}(i) = \mathbf{C}\mathbf{q}(i) + \mathbf{f}_s(i) \qquad (2)$$

where $\mathbf{q}(i) \in \mathbb{R}^n$ is the state vector, $\mathbf{u}(i) \in \mathbb{R}^r$ is the control input vector, $\mathbf{y}(i) \in \mathbb{R}^m$ is the measurement vector, $\mathbf{d}(i) \in \mathbb{R}^o$ is an unknown disturbance vector, $\mathbf{f}_a(i) \in \mathbb{R}^r$ is the actuator fault vector, $\mathbf{f}_s(i) \in \mathbb{R}^m$ is the sensor fault vector, and matrices $\mathbf{F} \in \mathbb{R}^{nxn}$, $\mathbf{G} \in \mathbb{R}^{nxr}$, $\mathbf{C} \in \mathbb{R}^{mxn}$ are known finite valued matrices.

For such a system (1), (2), which must be observable, the state estimator can be defined as

$$\mathbf{q}_e(i+1) = \mathbf{F}\mathbf{q}_e(i) + \mathbf{G}\mathbf{u}(i) + \mathbf{J}(\mathbf{y}(i) - \mathbf{y}_e(i)) \qquad (3)$$

$$\mathbf{y}_e(i) = \mathbf{C}\mathbf{q}_e(i) \qquad (4)$$

where $\mathbf{J} \in \mathbb{R}^{nxm}$ is the observer gain matrix. In the standard setup, the residual generator has the following form

$$\mathbf{r}(i) = \mathbf{V}_E(\mathbf{y}(i) - \mathbf{y}_e(i)) = \mathbf{V}_E(\mathbf{y}(i) - \mathbf{C}\mathbf{q}_e(i)) \qquad (5)$$

where $\mathbf{V}_E \in \mathbb{R}^{pxm}$ is a projection matrix and $\mathbf{r}(i) \in \mathbb{R}^p$ is the residual vector.

The actual prediction error and residuals can be written as

$$\mathbf{e}(i+1) = \mathbf{q}(i+1) - \mathbf{q}_e(i+1) = \mathbf{F}_e\mathbf{e}(i) + \mathbf{G}\mathbf{f}_a(i) - \mathbf{J}\mathbf{f}_s(i) + \mathbf{E}\mathbf{d}(i) \qquad (6)$$

$$\mathbf{r}(i) = \mathbf{V}_E\mathbf{C}\mathbf{e}(i) + \mathbf{V}_E\mathbf{f}_s(i) = \mathbf{H}\mathbf{e}(i) + \mathbf{V}_E\mathbf{f}_s(i) \qquad (7)$$

$$\mathbf{r}(i+1) = \mathbf{H}\mathbf{F}_e\mathbf{e}(i) + \mathbf{H}\mathbf{d}(i) + \mathbf{H}\mathbf{G}\mathbf{f}_a(i) - \mathbf{H}\mathbf{J}\mathbf{f}_s(i) + \mathbf{V}_E\mathbf{f}_s(i+1) \qquad (8)$$

where

$$\mathbf{F}_e = \mathbf{F} - \mathbf{J}\mathbf{C}, \qquad \mathbf{H} = \mathbf{V}_E\mathbf{C} \qquad (9)$$

The condition for (5) to be a residual generator is

$$\mathbf{H}\mathbf{E} = \mathbf{V}_E\mathbf{C}\mathbf{E} = \mathbf{0}, \qquad \mathbf{V}_E\mathbf{C} \neq \mathbf{0} \qquad (10)$$

together with stabile eigenvalues of the estimator system matrix (9). These conditions assure that, after a transient due to the effect of initial conditions, in the absence of faults the residual will be almost equal to zero.

2. Projection matrix

Consider that exist to (10) any solution of the form

$$\mathbf{YE} = \mathbf{XCE} \tag{11}$$

Pre-multiplying (10) from right-hand side by identity matrix leads to

$$\mathbf{YE} = \mathbf{YE}((\mathbf{CE})^T\mathbf{CE})^{-1}(\mathbf{CE})^T\mathbf{CE} = \mathbf{XCE} \tag{12}$$

$$\mathbf{X} = \mathbf{YE}((\mathbf{CE})^T\mathbf{CE})^{-1}(\mathbf{CE})^T = \mathbf{XCE}(\mathbf{CE})^+ \tag{13}$$

respectively, where

$$(\mathbf{CE})^+ = ((\mathbf{CE})^T\mathbf{CE})^{-1}(\mathbf{CE})^T \tag{14}$$

is the right pseudoinverse of $\mathbf{CE}$.

Pre-multiplying of (13) from right-hand side by CE gives

$$\mathbf{XCE} = \mathbf{XCE}(\mathbf{CE})^+\mathbf{CE} \tag{15}$$

$$\mathbf{X}(\mathbf{I}_m - \mathbf{CE}(\mathbf{CE})^+)\mathbf{CE} = \mathbf{0} \tag{16}$$

respectively. Thus, (16) implies, that projection matrix can be designed as follows

$$\mathbf{X}(\mathbf{I}_m - \mathbf{CE}(\mathbf{CE})^+)\mathbf{CE} = \mathbf{0} \tag{17}$$

where $\mathbf{X} \in \mathbb{R}^{pxm}$ is any arbitrary nonzero matrix and $\mathbf{I} \in \mathbb{R}^{mxm}$. Since maximal number of linear independent rows in $\mathbf{V}_E$ is m - rank $(\mathbf{CE})$, it is possible to set $p = m$ - rank $(\mathbf{CE})$ to design number of rows for $\mathbf{X}$.

3. Residual factorization

Assuming zero initial condition as well as the absence of faults, the $\mathcal{Z}$ transforms of (6) and (7) are

$$\tilde{\mathbf{e}}(z) = (z\mathbf{I} - \mathbf{F}_e)^{-1}\mathbf{E}\tilde{\mathbf{d}}(z) \tag{18}$$

$$\tilde{\mathbf{r}}(z) = \mathbf{H}(z\mathbf{I} - \mathbf{F}_e)^{-1}\mathbf{E}\tilde{\mathbf{d}}(z) = \mathbf{0} \tag{19}$$

If there are no multiple eigenvalues, eigenvalues factorization of $\mathbf{F}_e$ takes the form

$$\mathbf{F}_e = \mathbf{NZM}^T \tag{20}$$

where

$$\mathbf{M}^T\mathbf{N} = \mathbf{NM}^T = \mathbf{I} \tag{21}$$

$$\mathbf{N} = \begin{bmatrix} \mathbf{n}_1 & \mathbf{n}_2 & \cdots & \mathbf{n}_n \end{bmatrix} \tag{22}$$

$$\mathbf{M}^T = \begin{bmatrix} \mathbf{m}_1^T \\ \mathbf{m}_2^T \\ \vdots \\ \mathbf{m}_n^T \end{bmatrix}, \qquad \mathbf{Z} = \begin{bmatrix} z_1 & 0 & \cdots & 0 \\ 0 & z_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & z_n \end{bmatrix} \tag{23}$$

$\mathbf{N}$ is the right eigenvector matrix, $\mathbf{M}^T$ is the left eigenvector matrix, and $\varrho(\mathbf{F}_e) = \{z_l, \mid z_l < 1|, l = 1,2,...,n\}$ is the eigenvalues spectrum of $\mathbf{F}_e$.

Then, resolvent $\mathcal{Q}$ can be written as

$$\mathcal{Q} = (z\mathbf{I} - \mathbf{F}_e)^{-1} = (z\mathbf{NM}^T - \mathbf{NZM}^T)^{-1} = \mathbf{N}(z\mathbf{I} - \mathbf{Z})^{-1}\mathbf{M}^T \tag{24}$$

$$\mathcal{Q} = \sum_{l=1}^{n} \frac{\mathbf{n}_l\mathbf{m}_l^T}{z - z_l} \tag{25}$$

respectively. After substitution of (25) into (22) the residual factorization takes form

$$\tilde{\mathbf{r}}(z) = \sum_{l=1}^{n} \frac{\mathbf{Hn}_l\mathbf{m}_l^T\mathbf{E}}{z - z_l}\tilde{\mathbf{d}}(z) \tag{26}$$

4. Eigenstructure assignment

Given eigenvalues spectrum $\varrho(\mathbf{F}_e) = \{z_l, \mid z_l < 1|, l = 1,2,...,n\}$ then the necessary property for eigenstructure assignment can be written as

$$(\mathbf{F} - \mathbf{JC})\mathbf{n}_l = z_l\mathbf{n}_l, \quad l = 1,2,\ldots,n \tag{27}$$

$$\mathbf{JCn}_l = -(z_l\mathbf{I} - \mathbf{F})\mathbf{n}_l \tag{28}$$

respectively. If one apply (28) to $\mathbf{JC}$, the standard form of eigenstructure assignment can be formulated as

$$\mathbf{JC}\begin{bmatrix} \mathbf{n}_1 & \cdots & \mathbf{n}_n \end{bmatrix} = -\begin{bmatrix} (z_1\mathbf{I} - \mathbf{F})\mathbf{n}_1 & \cdots & (z_n\mathbf{I} - \mathbf{F})\mathbf{n}_n \end{bmatrix} \tag{29}$$

Using condition (26), disturbance decoupling can be obtained if each column of $\mathbf{E}$ is assigned as a right eigenvectors of $\mathbf{F}_e$, i.e.

$$\mathbf{JC}\begin{bmatrix} \mathbf{e}_1 & \cdots & \mathbf{e}_o \end{bmatrix} = -\begin{bmatrix} (z_1\mathbf{I} - \mathbf{F})\mathbf{e}_1 & \cdots & (z_o\mathbf{I} - \mathbf{F})\mathbf{e}_o \end{bmatrix} \tag{30}$$

where $o < n$ and

$$\mathbf{E} = \begin{bmatrix} \mathbf{e}_1 & \cdots & \mathbf{e}_o \end{bmatrix} \tag{31}$$

Analogously, the estimator output decoupling can be obtained, if each row of $\mathbf{H}$ is assigned as a left eigenvectors of $\mathbf{F}_e$. Since orthogonal property (21), the estimator output decoupling can be formulated with columns of matrix $\mathbf{H}^T$ assigned as right eigenvectors of $\mathbf{F}_e$, i.e.

$$\mathbf{JC}\begin{bmatrix} \mathbf{h}_1 & \cdots & \mathbf{h}_p \end{bmatrix} = -\begin{bmatrix} (z_1\mathbf{I} - \mathbf{F})\mathbf{h}_1 & \cdots & (z_p\mathbf{I} - \mathbf{F})\mathbf{h}_p \end{bmatrix} \tag{32}$$

where $p < n$ and

$$\mathbf{H}^T = \begin{bmatrix} \mathbf{h}_1 & \cdots & \mathbf{h}_p \end{bmatrix} \tag{33}$$

Using (30) and (32) the mixture of disturbance decoupling and partly output decoupling can be formulated as follows

$$\mathbf{JCK} = -\mathbf{L} \tag{34}$$

where

$$\mathcal{H} = \{\mathbf{h}_l^\circ, \quad l = 1,2,\ldots q, \quad q \le p, \quad o + q < n\} \tag{35}$$

is the set of selected columns of $\mathbf{H}^T$, and

$$\mathbf{L} = \mathbf{L}_E + \mathbf{L}_H \tag{36}$$

$$\mathbf{L}_E = -\begin{bmatrix} (z_1\mathbf{I} - \mathbf{F})\mathbf{e}_1 & \cdots & (z_o\mathbf{I} - \mathbf{F})\mathbf{e}_o \end{bmatrix} \tag{37}$$

$$\mathbf{L}_H = -\begin{bmatrix} (z_1\mathbf{I} - \mathbf{F})\mathbf{h}_1^\circ & \cdots & (z_q\mathbf{I} - \mathbf{F})\mathbf{h}_q^\circ \end{bmatrix} \tag{38}$$

$$\mathbf{K} = \begin{bmatrix} \mathbf{e}_1 & \cdots & \mathbf{e}_o & \mathbf{h}_1^\circ & \cdots & \mathbf{h}_q^\circ \end{bmatrix} \tag{39}$$

5. Gain matrix design

Pre-multiplying (34) from right-hand side by identity matrix leads to

$$\mathbf{JCK} = -\mathbf{L}((\mathbf{CK})^T\mathbf{CK})^{-1}(\mathbf{CK})^T\mathbf{CK} \tag{40}$$

$$\mathbf{J} = -\mathbf{L}((\mathbf{CK})^T\mathbf{CK})^{-1}(\mathbf{CK})^T = -\mathbf{L}(\mathbf{CK})^+ \tag{41}$$

respectively, where

$$(\mathbf{CK})^+ = ((\mathbf{CK})^T\mathbf{CK})^{-1}(\mathbf{CK})^T \tag{42}$$

is the right pseudoinverse of $\mathbf{CK}$.

According to (41), (42) the relation holds

$$(\mathbf{J}+\mathbf{L}(\mathbf{CK})^{+}\mathbf{CK}+\mathbf{0}=\mathbf{0} \quad (43)$$

Matrix $\mathbf{0}$ on the left-hand side of (43) represents any trivial solution from the null-space of $\mathbf{CK}$, i.e.

$$\mathbf{Z}(\mathbf{I}_m-\mathbf{CK}(\mathbf{CK})^{+})=\mathbf{0} \quad (44)$$

where $\mathbf{Z}$ is an arbitrary nonzero matrix of appropriate dimension.

Thus, the general form of the solution (41) is

$$\mathbf{J}=-\mathbf{L}(\mathbf{CK})^{+}+\mathbf{Z}(\mathbf{I}_m-\mathbf{CK}(\mathbf{CK})^{+}) \quad (45)$$

One can prove obtain any solution of (34) if $o + q = n$ ($\mathbf{K}$ and $\mathbf{L}$ are square matrices).

6. Illustrative example

To demonstrate properties one simple system described by the discrete-time state-space equations (1), (2) was considered where

$$\mathbf{F}=\begin{bmatrix}0.9993 & 0.0987 & 0.0042\\ 0.0212 & 0.9612 & 0.0775\\ 0.3985 & 0.7187 & 0.5737\end{bmatrix},\quad \mathbf{C}=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 1\end{bmatrix}$$

$$\mathbf{G}=\begin{bmatrix}0.0051 & 0.0050\\ 0.1029 & 0.9612\\ 0.0387 & 0.5737\end{bmatrix},\quad \mathbf{E}=\begin{bmatrix}2.9345\\ 1.9764\\ 3.9234\end{bmatrix}$$

and (9) computing with Matlab Control Toolbox yields for given system matrices

$$\mathbf{H}=\begin{bmatrix}0.4253 & -0.4944 & -0.0691\\ -0.4944 & 0.5747 & 0.0803\end{bmatrix}$$

so that the desired left eigenvector was chosen as

$$\mathbf{h}^{\circ T}=\mathbf{h}_2^T=\begin{bmatrix}-0.4944 & 0.5747 & 0.0803\end{bmatrix}$$

with correspondence to desired eigenvalue $z_H = 0.2$. Together with desired eigenvalue $z_E = 0.1$ were constructed matrices

$$\mathbf{K}=\begin{bmatrix}\mathbf{e} & \mathbf{h}^{\circ}\end{bmatrix}=\begin{bmatrix}2.9345 & -0.4944\\ 1.9764 & 0.5747\\ 3.9234 & 0.0803\end{bmatrix}$$

$$\mathbf{L}=\begin{bmatrix}(0.1\mathbf{I}-\mathbf{F})\mathbf{e} & (0.2\mathbf{I}-\mathbf{F})\mathbf{h}^{\circ}\end{bmatrix}=\begin{bmatrix}-2.8505 & 0.3381\\ -2.0684 & -0.4332\\ -4.4161 & -0.2514\end{bmatrix}$$

Upon some computation was found

$$\mathbf{J}=\begin{bmatrix}0.5569 & -0.1641\\ -0.1732 & 0.5519\\ 0.2032 & 0.5124\end{bmatrix},\quad \mathbf{N}=\begin{bmatrix}-0.6901 & 0.5555 & 0.6485\\ -0.5713 & 0.3741 & -0.7359\\ -0.4442 & 0.7426 & -0.1053\end{bmatrix}$$

and associated eigenvalues spectrum $\varrho(\mathbf{F}_e)=\{0.41, 0.1, 0.2\}$. One can verify the decoupling property, where

$$\mathbf{Ne}=\begin{bmatrix}-4.8972\\ 5.2830\\ 0.0000\end{bmatrix},\quad \mathbf{h}_2^T\mathbf{N}=\begin{bmatrix}-0.0228 & 0.0000 & -0.7623\end{bmatrix}$$

Concluding remarks

The paper presents the basic design principle of a robust residual generation for discrete-time linear multi-input and multi-output (MIMO) dynamic systems. The exposed problems of disturbance decoupling, in estimator based residuals was extended to (partly) output decoupling. Presented extensions use the eigenvector matrix properties and combine two standard design formulations. A constructive procedure for finding the estimator gain matrix and an illustrative example of the results are given. Presented applications can be considered as a task concerned the class of eigenstructure assignment problems.

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