Pole assignment in robust state observer design

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Abstract
In the paper a design procedure generally known as regional pole assignment is applied to discrete-time uncertain system state observer design. The standard form of the observer gain matrix is presented to obtain design algorithm based on the discrete algebraic Riccati equation, as well as is derived the linear matrix inequalities reformulation of the design algorithm. The design results in the assignment of the observer state matrix eigenvalues to any desired circular location in stabile Z-plane circle. An example is given to illustrate the algorithm performance.

Keywords: Pole assignment, robust state estimation, discrete-time systems.

Introduction

The state-space control system design assumes that the full system state vector is available. For most control system the measurement of the full state vector is impractical, if not impossible, in all but the simplest of systems and a technique for estimating the states or outputs of a plant from the available plant information is proposed. The systems that estimate the states of another system are generally known as state observers. Using in control, the inclusion of a state observer into control loop can reduce the robustness of state feedback control, where, specifically, system matrix disturbance can prevent the convergence of observer estimates.

It is well known that the transient response of a linear system is related to the location of its poles (the system matrix eigenvalues), and therefore a design procedure generally known as pole assignment is mostly used in design controllers, as well as state observers for fixed linear discrete-time, as well as continuous-time systems. Because of the parameter uncertainty is difficult to assign eigenvalues at exact stabile location but in many occasions it is enough to assign them in a specified region for some practical design specification.

Equations describing observers can be developed in several different ways, here is chosen to take in develop the state-space approach. This approach is based on the controlled model of the system, where control model input is the difference between actual output and estimated output of the system. However, some results on state observer design based on robust eigenvalues and eigenvectors assignment are available in [3], [6] but almost given by general inverse matrices.

The paper presents a robust algorithm to assign all eigenvalues of the discrete-time state observer system matrix $\mathbf{F}_e$ in a specified disk with radius ρ and center $a = a$+j0 lying within the unit circle in the complex $\mathcal{Z}$ plain. The design criterion is quadratic stabilization of the state estimation error system under an observer gain matrix using Lyapunov function with substitution modification for regional eigenvalues assignment. The paper considers a similar problem of robust pole assignment as was given in [7] but presents more simpler form to derive the design algorithms and LMI reformulation.

1. Robust observer design

Specifically, a discrete-time uncertain system can be considered as

$$\mathbf{q}(i+1) = (\mathbf{F} + \Delta\mathbf{F})\mathbf{q}(i) + \mathbf{G}\mathbf{u}(i) \tag{1}$$

$$\mathbf{y}(i) = \mathbf{C}\mathbf{q}(i) \tag{2}$$

$\mathbf{q}(i) \in \mathbb{R}^n$, $\mathbf{u}(i) \in \mathbb{R}^r$, $\mathbf{y}(i) \in \mathbb{R}^m$, matrices $\mathbf{F} \in \mathbb{R}^{nxn}$, $\mathbf{G} \in \mathbb{R}^{nxr}$, $\mathbf{C} \in \mathbb{R}^{mxn}$ are finite valued, and $\Delta\mathbf{F} \in \mathbb{R}^{nxn}$ is unknown matrix which represents time-varying parametric uncertainties.

It is assumed that considered uncertainty matrix is norm bounded and can be written as

$$\Delta\mathbf{F} = \mathbf{N}\mathbf{H}\mathbf{M} \tag{3}$$

$$\mathbf{H}^T\mathbf{H} \le \mathbf{I} \tag{4}$$

where $\mathbf{N} \in \mathbb{R}^{lxn}$, $\mathbf{M} \in \mathbb{R}^{nxl}$ are known constant matrices which define the structure of the uncertainty and the parameter uncertainty $\mathbf{H}$ belong to the set

$$\mathcal{U} = \{\mathbf{H} \in \mathbb{R}^{nxl} : \mathbf{H}^T\mathbf{H} \le \mathbf{I}\} \tag{5}$$

For such a system (1), (2), which must be quadratically stabilizable and detectable, the robust state observer is defined as

$$\mathbf{q}_e(i+1) = (\mathbf{F} + \Delta\mathbf{F})\mathbf{q}_e(i) + \mathbf{G}\mathbf{u}(i) + \mathbf{J}(\mathbf{y}_e(i) - \mathbf{y}(i)) \tag{6}$$

$$\mathbf{y}_e(i) = \mathbf{C}\mathbf{q}_e(i) \tag{7}$$

where $\mathbf{J} \in \mathbb{R}^{nxm}$ is the observer gain matrix. Substituting (7) into (6) gives

$$\mathbf{q}_e(i+1) = (\mathbf{F} + \Delta\mathbf{F} + \mathbf{J}\mathbf{C})\mathbf{q}_e(i) + \mathbf{G}\mathbf{u}(i) - \mathbf{J}\mathbf{y}(i) \tag{8}$$

where

$$\mathbf{F}_e = \mathbf{F} + \Delta\mathbf{F} + \mathbf{J}\mathbf{C} \tag{9}$$

is the observer system matrix. Then it is obvious, that the state estimation error system is

$$\mathbf{e}(i+1) = (\mathbf{F} + \Delta\mathbf{F} + \mathbf{J}\mathbf{C})\mathbf{e}(i) \tag{10}$$

and $\mathbf{e}(i) = \mathbf{q}(i) - \mathbf{q}_e(i)$.

2. Quadratic stabilization

The state estimation error system (10) is quadratically stabilizable, if there exists a positive definite symmetric matrix $\mathbf{P} \in \mathbb{R}^{nxn}$ such that

$$v(\mathbf{e}(i)) = \mathbf{e}^T(i)\mathbf{P}\mathbf{e}(i) > 0 \tag{11}$$

$$\Delta v(\mathbf{e}(i)) = \mathbf{e}^T(i+1)\mathbf{P}\mathbf{e}(i+1) - \mathbf{e}^T(i)\mathbf{P}\mathbf{e}(i) < 0 \tag{12}$$

are the Lyapunov function and its difference, respectively. Substituting (10) into (12) implies that

$$(\mathbf{F} + \Delta\mathbf{F} + \mathbf{JC})^T \mathbf{P}(\mathbf{F} + \Delta\mathbf{F} + \mathbf{JC}) - \mathbf{P} < 0 \tag{13}$$

for all $\mathbf{H} \in \mathcal{U}$. From point of Schur complement, (12) is then equivalent to formula (* denotes symmetric transposition)

$$\begin{bmatrix} -\mathbf{P}^{-1} & \mathbf{F} + \Delta\mathbf{F} + \mathbf{JC} \\ * & -\mathbf{P} \end{bmatrix} = \\ = \begin{bmatrix} -\mathbf{P}^{-1} & \mathbf{F} \\ * & -\mathbf{P} \end{bmatrix} + \begin{bmatrix} \mathbf{0} & \mathbf{JC} \\ * & \mathbf{0} \end{bmatrix} + \begin{bmatrix} \mathbf{0} & \Delta\mathbf{F} \\ * & \mathbf{0} \end{bmatrix} < 0 \tag{14}$$

$$\begin{bmatrix} -\mathbf{P}^{-1} & \mathbf{F} + \Delta\mathbf{F} + \mathbf{JC} \\ * & -\mathbf{P} \end{bmatrix} = \\ = \begin{bmatrix} -\mathbf{P}^{-1} & \mathbf{F} \\ * & -\mathbf{P} \end{bmatrix} - \begin{bmatrix} -\mathbf{I} & -\mathbf{JC} \\ * & -\mathbf{I} \end{bmatrix} + \begin{bmatrix} -\mathbf{I} & \Delta\mathbf{F} \\ * & -\mathbf{I} \end{bmatrix} < 0 \tag{15}$$

respectively. As all terms of (15) are symmetric matrices, by Schur complement two last of terms can be rewritten as

$$\mathbf{X} = -(-\mathbf{I} + \mathbf{C}^T\mathbf{J}^T\mathbf{JC}) + (-\mathbf{I} + \mathbf{M}^T\mathbf{H}^T\mathbf{N}^T\mathbf{NHM}) = \\ = -\mathbf{C}^T\mathbf{J}^T\mathbf{JC} + \mathbf{M}^T\mathbf{H}^T\mathbf{N}^T\mathbf{NHM} \tag{16}$$

It is straightforward to verify that for $\mathbf{I} > \mathbf{A}$ yields

$$\mathbf{AA}^T - \mathbf{A} - \mathbf{A}^T + \mathbf{I} = (\mathbf{I} - \mathbf{A})(\mathbf{I} - \mathbf{A})^T \tag{17}$$

$$\mathbf{AA}^T = \mathbf{A} + \mathbf{A}^T - (\mathbf{I} - (\mathbf{I} - \mathbf{A})(\mathbf{I} - \mathbf{A})^T) \leq \mathbf{A} + \mathbf{A}^T \tag{18}$$

Then, using (4) and (18), as well as the matrix identity

$$\mathbf{BC}^T + \mathbf{CB}^T \leq \mathbf{BB}^T + \mathbf{CC}^T \tag{19}$$

(16) may be rewritten as

$$\mathbf{X} \leq -\mathbf{C}^T\mathbf{R}^{-1}\mathbf{C} + \mathbf{M}^T\mathbf{H}^T\mathbf{N}^T + \mathbf{NHM} \leq \\ \leq -\mathbf{C}^T\mathbf{R}^{-1}\mathbf{C} + \tfrac{1}{\varepsilon}\mathbf{NN}^T + \varepsilon\mathbf{M}^T\mathbf{H}^T\mathbf{HM} \leq \\ \leq -\mathbf{C}^T\mathbf{R}^T\mathbf{C} + \tfrac{1}{\varepsilon}\mathbf{NN}^T + \varepsilon\mathbf{M}^T\mathbf{M} \tag{20}$$

where $\mathbf{R}^{-1} = \mathbf{J}^T\mathbf{J} \geq 0$ and $\varepsilon > 0$.

Using Schur complement notation for (20) inequality (15) takes the form

$$\begin{bmatrix} -\mathbf{P}^{-1} & \mathbf{F} + \Delta\mathbf{F} + \mathbf{JC} \\ * & -\mathbf{P} \end{bmatrix} \leq \\ = \begin{bmatrix} -\mathbf{P}^{-1} & \mathbf{F} \\ * & -\mathbf{P} \end{bmatrix} - \begin{bmatrix} \tfrac{1}{\varepsilon}\mathbf{NN}^T & \mathbf{0} \\ * & -\mathbf{C}^T\mathbf{R}^T\mathbf{C} + \varepsilon\mathbf{M}^T\mathbf{M} \end{bmatrix} < 0 \tag{21}$$

That is

$$\begin{bmatrix} -\mathbf{P}^{-1} + \tfrac{1}{\varepsilon}\mathbf{NN}^T & \mathbf{F} \\ * & -\mathbf{P} - \mathbf{C}^T\mathbf{R}^T\mathbf{C} + \varepsilon\mathbf{M}^T\mathbf{M} \end{bmatrix} < 0 \tag{22}$$

$$\mathbf{F}(\mathbf{P} + \mathbf{C}^T\mathbf{R}^{-1}\mathbf{C} - \varepsilon\mathbf{M}^T\mathbf{M})\mathbf{F}^T - \mathbf{P}^{-1} + \tfrac{1}{\varepsilon}\mathbf{NN}^T < 0 \tag{23}$$

respectively. Thus, previous inequality implies that for a positive definite symmetric matrix $\mathbf{Q}$ there is $\mathbf{P}$ as a solution of the Riccati equation

$$\mathbf{F}(\mathbf{P} + \mathbf{C}^T\mathbf{R}^{-1}\mathbf{C} - \varepsilon\mathbf{M}^T\mathbf{M})\mathbf{F}^T - \mathbf{P}^{-1} + \tfrac{1}{\varepsilon}\mathbf{NN}^T + \mathbf{Q} = 0 \tag{24}$$

conditioned by inequality

$$\mathbf{P} - \varepsilon\mathbf{M}^T\mathbf{M} > 0 \tag{25}$$

3. Observer gain matrix

Denote

$$\mathbf{F}_J = \mathbf{F} + \mathbf{JC} \tag{26}$$

and consider the Lyapunov function of the estimated error system in the form

$$\mathbf{V} = (\mathbf{F}_J + \Delta\mathbf{F})\mathbf{P}^{-1}(\mathbf{F}_J + \Delta\mathbf{F})^T - \mathbf{P}^{-1} < 0 \tag{27}$$

Since (13) implies (23), analogously (27) implies

$$\mathbf{V} \leq \mathbf{F}_J(\mathbf{P} - \varepsilon\mathbf{M}^T\mathbf{M})^{-1}\mathbf{F}_J^T - \mathbf{P}^{-1} + \varepsilon^{-1}\mathbf{NN}^T < 0 \tag{28}$$

Using (24) one can obtain

$$\mathbf{F}(\mathbf{P} + \mathbf{C}^T\mathbf{R}^{-1}\mathbf{C} - \varepsilon\mathbf{M}^T\mathbf{M})^{-1}\mathbf{F}^T + \varepsilon^{-1}\mathbf{NN}^T + \mathbf{Q} = \mathbf{P}^{-1} \tag{29}$$

and substituting (29) into (28) results

$$\mathbf{V} \leq \mathbf{F}_J(\mathbf{P} - \varepsilon\mathbf{M}^T\mathbf{M})^{-1}\mathbf{F}_J^T - \mathbf{F}(\mathbf{P} + \mathbf{C}^T\mathbf{R}^{-1}\mathbf{C} - \varepsilon\mathbf{M}^T\mathbf{M})^{-1}\mathbf{F}^T - \mathbf{Q} \tag{30}$$

$$\mathbf{V} \leq \mathbf{F}_J(\mathbf{P} - \varepsilon\mathbf{M}^T\mathbf{M})^{-1}\mathbf{F}_J^T - \mathbf{DF}^T - \mathbf{Q} \tag{31}$$

respectively, where

$$\mathbf{D} = \mathbf{F}(\mathbf{P} + \mathbf{C}^T\mathbf{R}^{-1}\mathbf{C} - \varepsilon\mathbf{M}^T\mathbf{M})^{-1} \tag{32}$$

and

$$\mathbf{F} = \mathbf{D}(\mathbf{P} + \mathbf{C}^T\mathbf{R}^{-1}\mathbf{C} - \varepsilon\mathbf{M}^T\mathbf{M}) \tag{33}$$

Using (33) and denoting

$$\mathbf{J} = \mathbf{DY} \tag{34}$$

inequality (31) can be rewritten as

$$\mathbf{V} \leq \mathbf{D}(\mathbf{P} + \mathbf{C}^T\mathbf{R}^{-1}\mathbf{C} - \varepsilon\mathbf{M}^T\mathbf{M} + \mathbf{YC})(\mathbf{P} - \varepsilon\mathbf{M}^T\mathbf{M})^{-1}\mathbf{F}_C^T = \\ = \mathbf{F}_C(\mathbf{P} - \varepsilon\mathbf{M}^T\mathbf{M})^{-1}\mathbf{F}_C^T \tag{35}$$

where

$$\mathbf{F}_C = \mathbf{D}(\mathbf{P} + \mathbf{C}^T\mathbf{R}^{-1}\mathbf{C} - \varepsilon\mathbf{M}^T\mathbf{M} + \mathbf{YC}) \tag{36}$$

Taking

$$\mathbf{C}^T\mathbf{R}^{-1}\mathbf{C} + \mathbf{YC} = \mathbf{0} \tag{37}$$

the observer gain matrix $\mathbf{J}$ is given by equation

$$\mathbf{J} = \mathbf{DY} = -\mathbf{DC}^T\mathbf{R}^{-1} \tag{38}$$

and inequality (35) takes form

$$\mathbf{V} \leq \mathbf{D}(\mathbf{P} - \varepsilon\mathbf{M}^T\mathbf{M})\mathbf{D}^T - \mathbf{Q} - \mathbf{D}(\mathbf{P} - \varepsilon\mathbf{M}^T\mathbf{M})\mathbf{D}^T - \mathbf{DC}^T\mathbf{R}^{-1}\mathbf{CD}^T = \\ = -\mathbf{Q} - \mathbf{DC}^T\mathbf{R}^{-1}\mathbf{CD}^T = -\mathbf{Q} - \mathbf{JRJ}^T < 0 \tag{39}$$

Inequality (39) implies, that matrix $\mathbf{V}$ is negative definite.

4. Design based on LMI formalism

The procedure for deriving the optimization conditions yields, after using nontrivial transformations, a set of linear matrix inequalities (LMI).

Pre-multiplying (39) from left and right hand side by matrix $\mathbf{P} > 0$ gives

$$\mathbf{PF}(\mathbf{P}+\mathbf{C}^T\mathbf{R}^{-1}\mathbf{C}-\varepsilon\mathbf{M}^T\mathbf{M})\mathbf{F}^T\mathbf{P}-\mathbf{P}+\tfrac{1}{\varepsilon}\mathbf{PNN}^T\mathbf{P}<0 \qquad (40)$$

and LMIs for (40) can be written as

$$\mathbf{P}=\mathbf{P}^T>0 \qquad (41)$$

$$\begin{bmatrix} -\mathbf{P} & \mathbf{PN} & \mathbf{PF} \\ * & \mathbf{I} & \mathbf{0} \\ * & * & -\mathbf{P}-\mathbf{C}^T\mathbf{R}^{-1}\mathbf{C}+\varepsilon\mathbf{M}^T\mathbf{M} \end{bmatrix}<0 \qquad (42)$$

Since (20) can be written as

$$-\mathbf{P}+\varepsilon\mathbf{M}^T\varepsilon^{-1}\mathbf{I}\varepsilon\mathbf{M}<0 \qquad (43)$$

inequality (43) implies

$$\begin{bmatrix} -\mathbf{P} & \varepsilon\mathbf{M}^T \\ * & -\varepsilon\mathbf{I} \end{bmatrix}<0 \qquad (44)$$

The observer gain matrix is then given by equation

$$\mathbf{J}=-\mathbf{F}(\mathbf{P}+\mathbf{C}^T\mathbf{R}^{-1}\mathbf{C}-\varepsilon\mathbf{M}^T\mathbf{M})^{-1}\mathbf{C}^T\mathbf{R}^{-1} \qquad (45)$$

It is straightforward to verify that in nominal system case the state estimation error system (10) is stabilizable under an observer gain matrix $\mathbf{J}$ if exists a positive definite symmetric matrix $\mathbf{P}$ satisfying linear matrix inequality

$$\begin{bmatrix} -\mathbf{P} & \mathbf{PF} \\ * & -\mathbf{P}-\mathbf{C}^T\mathbf{R}^{-1}\mathbf{C} \end{bmatrix}<0 \qquad (46)$$

The observer gain matrix $\mathbf{J}$ for nominal conditions is given as

$$\mathbf{J}=-\mathbf{F}(\mathbf{P}+\mathbf{C}^T\mathbf{R}^{-1}\mathbf{C})^{-1}\mathbf{C}^T\mathbf{R}^{-1} \qquad (47)$$

5. Regional pole assignment

Consider a stabile linear discrete-time system described by the dual state-space equation

$$\mathbf{p}(i+1)=\tfrac{(\mathbf{F}-a\mathbf{I})^T}{\varrho}\mathbf{p}(i)+\tfrac{\mathbf{C}^T}{\varrho}\mathbf{u}(i) \qquad (48)$$

0< a <1, 0 < ϱ <1, a,$\varrho \in \mathbb{R}$, $\varrho \le$ a. If $\mathbf{P}$ be a symmetric positive definite matrix then there exists Lyapunov function $v(\mathbf{p}(i))$,

$$v(\mathbf{p}(i))=\mathbf{p}^T(i)\mathbf{P}\mathbf{p}(i) \qquad (49)$$

such that

$$\Delta v(\mathbf{p}(i))=\mathbf{p}^T(i+1)\mathbf{P}\mathbf{p}(i+1)-\mathbf{p}^T(i)\mathbf{P}\mathbf{p}(i)<0 \qquad (50)$$

Therefore if (50) holds, clearly

$$\mathbf{p}^T(i)\left(\frac{(\mathbf{F}-a\mathbf{I})}{\rho}\mathbf{P}\frac{(\mathbf{F}-a\mathbf{I})^T}{\rho}-\mathbf{P}\right)\mathbf{p}(i)<0 \qquad (51)$$

$$\frac{(\mathbf{F}-a\mathbf{I})}{\rho}\mathbf{P}\frac{(\mathbf{F}-a\mathbf{I})^T}{\rho}-\mathbf{P}<-\frac{\mathbf{Q}}{\rho^2} \qquad (52)$$

respectively, where $\mathbf{Q}$ is a symmetric positive definite matrix. Then non-equality (52) can be rewritten as

$$-a\mathbf{FP}-a\mathbf{PF}^T+\mathbf{FPF}^T+(a^2-\rho^2)\mathbf{P}<-\mathbf{Q} \qquad (53)$$

If z_h is an eigenvalue of $\mathbf{F}$, vector $\mathbf{m}_h^T$ is associated left eigenvector, z_h^* is the complex conjugated eigenvalue with z_h and associated complex conjugated left eigenvector is $\mathbf{m}_h^{*T}$, then holds

$$\mathbf{m}_h^T\mathbf{F}=z_h\mathbf{m}_h^T, \qquad \mathbf{F}^T\mathbf{m}_h^*=z_h^*\mathbf{m}_h^T \qquad (54)$$

Pre-multiplying (53) from left-hand side by $\mathbf{m}_h^T$ as well as from right-hand side by $\mathbf{m}_h^*$ give

$$-a\mathbf{m}_h^T\mathbf{FPm}_h^*-a\mathbf{m}_h^T\mathbf{PFm}_h^*+\mathbf{m}_h^T\mathbf{FPFm}_h^*+(a^2-\rho^2)\mathbf{m}_h^T\mathbf{Pm}_h^*< \\ <-\mathbf{m}_h^T\mathbf{Qm}_h^* \qquad (55)$$

and substituting (54) into (55) yields

$$(-a(z_h+z_h^*)+z_hz_h^*+(a^2-\rho^2))\mathbf{m}_h^T\mathbf{Pm}_h^*<-\mathbf{m}_h^T\mathbf{Qm}_h^* \qquad (56)$$

Since $\mathbf{Q}$ is positive definite, the positivity of $\mathbf{P}$ yields

$$-a(z_h+z_h^*)+z_hz_h^*+a^2-\rho^2<0 \qquad (57)$$

If $z_h = z_{hR} + \mathrm{j}z_{hI} = x + \mathrm{j}y$, then using it in (57) one can obtain

$$-2ax+x^2+y^2+a^2-\rho^2<0 \qquad (58)$$

$$(x-a)^2+y^2<\rho^2 \qquad (59)$$

respectively, which means that all eigenvalues of $\mathbf{F}$ are located in a specified disk with radius ϱ and center $a = a+\mathrm{j}0$ lying within the unit circle in the complex $\mathcal{Z}$ plain.

The existence condition of $\mathbf{P}$ satisfying (39) is equivalent to the existence of $\mathbf{P}^{-1}$ satisfying (27). Substituting

$$\mathbf{F}\leftarrow\frac{\mathbf{F}-a\mathbf{I}}{\rho}, \quad \mathbf{JC}\leftarrow\mathbf{J}\frac{\mathbf{C}}{\rho}, \quad \Delta\mathbf{F}\leftarrow\frac{\Delta\mathbf{F}}{\rho} \qquad (60)$$

into (27) gives

$$\mathbf{V}=\frac{(\mathbf{F}_e-a\mathbf{I})}{\rho}\mathbf{P}^{-1}\frac{(\mathbf{F}_e-a\mathbf{I})^T}{\rho}-\mathbf{P}^{-1}<0 \qquad (61)$$

where

$$\mathbf{F}_e=\mathbf{F}+\Delta\mathbf{F}+\mathbf{JC} \qquad (62)$$

$$\frac{\Delta\mathbf{F}}{\rho}=\frac{\mathbf{N}}{\sqrt{\rho}}\mathbf{H}\frac{\mathbf{M}}{\sqrt{\rho}} \qquad (63)$$

It is evident from (51) and (61) that using substitutions (60) all eigenvalues of $\mathbf{F}_e$ be located in a specified disk with radius ϱ and center $a = a+\mathrm{j}0$ lying within the unit circle in the complex $\mathcal{Z}$ plain.

6. Illustrative example

The dynamic system is described by the discrete-time state-space equations (1), (2), and (3) where

$$\mathbf{F}=\begin{bmatrix} 0.9993 & 0.0987 & 0.0042 \\ 0.0212 & 0.9612 & 0.0775 \\ 0.3985 & 0.7187 & 0.5737 \end{bmatrix}, \quad \mathbf{C}=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$$\mathbf{G}=\begin{bmatrix} 0.0051 & 0.0050 \\ 0.1029 & 0.9612 \\ 0.0387 & 0.5737 \end{bmatrix}, \quad \mathbf{N}=\begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad \mathbf{M}=0.2\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

Assuming the precision matrix

$$\mathbf{R}^{-1}=\begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$$

the optimal values of LMI variables are

$$\mathbf{P}=\begin{bmatrix} 13.4311 & 0.5218 & -2.3960 \\ 0.5218 & 11.0708 & -2.9650 \\ -2.3960 & -2.9650 & 12.8394 \end{bmatrix}, \quad \varepsilon=21.9711$$

and according to (45), the observer gain matrix $\mathbf{J}$ and error system matrix $\mathbf{F}_e = \mathbf{F} - \mathbf{JC}$ were obtained as follows

$$\mathbf{J} = \begin{bmatrix} -0.3412 & 0.0492 \\ 0.0543 & -0.4217 \\ -0.2159 & -0.4121 \end{bmatrix}, \quad \mathbf{F}_e = \begin{bmatrix} 0.6581 & 0.1479 & -0.2879 \\ 0.0755 & 0.5395 & -0.2899 \\ 0.1716 & 0.3066 & -0.0543 \end{bmatrix}$$

with error system matrix eigenvalues spectrum

$$\rho(\mathbf{F}_e) = \{0.5800, 0.2816 \pm \mathrm{j}0.1054\}$$

Based on the same performance specification as mentioned above, using regional eigenvalues placement parameters a = 0.3, ϱ = 0.3, as well as substitutions (60), (63), the optimal solution of LMIs gives

$$\mathbf{P} = \begin{bmatrix} 50.2135 & 15.9235 & -33.6428 \\ 15.9235 & 46.0448 & -37.5997 \\ -33.6428 & -37.5997 & 65.7243 \end{bmatrix}, \quad \varepsilon = 169.0870$$

The observer gain matrix and error system matrix, using the same substitutions, was computed as follows

$$\mathbf{J} = \begin{bmatrix} -0.3814 & 0.0947 \\ 0.0819 & -0.4989 \\ -0.1724 & -0.5214 \end{bmatrix}, \quad \mathbf{F}_e = \begin{bmatrix} 0.6179 & 0.1934 & -0.2826 \\ 0.1031 & 0.4623 & -0.3395 \\ 0.2151 & 0.1973 & -0.1201 \end{bmatrix}$$

and obtained error system matrix eigenvalues spectrum is

$$\rho(\mathbf{F}_e) = \{0.0993, 0.3468, 0.5140\}$$

Concluding remarks

The paper presents a robust algorithm to assign all eigenvalues of the discrete-time state observer system matrix $\mathbf{F}_e$ in a specified disk with radius ϱ and center $a = a + \mathrm{j}0$ lying within the unit circle in the complex $\mathcal{Z}$ plain. These parameters of the desired disk are just need to be specified, and the pole locations within disk are only dependent on the matrices $\mathbf{Q}$ and $\mathbf{R}$. The design conditions are expressed by linear matrix inequalities, as well by discrete Riccati equation, for systems with parameter uncertainty in system matrix $\mathbf{F}$. Used method presents some simplification of design features, where simulation results have confirmed the effectiveness of the suggested approach.

Acknowledgements

The work presented in this paper was supported by Grant Agency of Ministry of Education and Academy of Science of Slovak Republic VEGA under Grant No. 1/2173/05.

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