Robust control synthesis of benchmark casting process temperature fields

Cyril Belavý, Pavol Buček, Pavol Noga, Gabriel Hulkó

Abstract
In the paper simulation of a robust control of temperature fields of the mould at casting process is presented. Temperature fields of the mould were modelled and studied using a finite element method based software package ProCAST and numerical models in the form of a lumped-input/distributed-output system were obtained. For the robust control synthesis with an internal model control structure, a model uncertainty of the controlled system was considered. Simulation was realized using the Distributed Parameter Systems Blockset for MATLAB & Simulink.

Keywords: robust control, distributed parameter system (DPS), lumped-input/ distributed-output system (LDS), finite element method (FEM), internal model control (IMC), casting process

Introduction

Nowadays is at disposal a number of software products, environments and virtual try-out spaces for numerical dynamical analysis of machines and processes practically in all engineering disciplines. Most of the analyzed dynamical systems, including of the casting processes, in fact are distributed parameter systems given by the numerical structures on complex-shape 3D definition domains. Dynamical characteristics obtained by these numerical methods offer wide possibilities for control of systems as distributed parameter systems.

Temperature control in the casting mould can have considerable implications for the elimination of defects that may arise in the process. For analysis of casting process dynamics as DPS, benchmark of the casting processes was designed. It has to be able to embrace typical effects and critical moment of casting. Temperature fields of the mould at casting were modelled and studied using ProCAST, which is a FEM based software package. For control synthesis purpose, numerical models in the form of a lumped-input/distributed-output system were made up by means of finite element analysis.

Based on uncertainty analysis of the models, the robust control synthesis has been done. Simulation of the robust control of temperature fields was realized using the Distributed Parameter Systems Blockset for MATLAB & Simulink, a third-party software product of The MathWorks, Inc., developed at the Department of Automation, Informatics and Instrumentation, Faculty of Mechanical Engineering STU (Hulkó 2004).

1. LDS representation of DPS

In general, DPS are systems whose state or output variables, *X*(*x,y,z,t*)/*Y*(*x,y,z,t*) are distributed variables or fields of variables, where (*x,y,z*) is a vector in 3D. These systems are often considered as systems whose dynamics is described by partial differential equations (PDE), (Butkovskij 1965), (Lions 1971). In the input-output relation, PDE define distributed-input/distributed-output systems (DDS) between distributed input *U*(*x,y,z,t*) and distributed output variables *Y*(*x,y,z,t*), at initial and boundary conditions given. Distributed parameter systems frequently are found in the engineering practice as LDS, (Hulkó 1987, 1998) see Fig. 1, having the structure according Fig. 2.

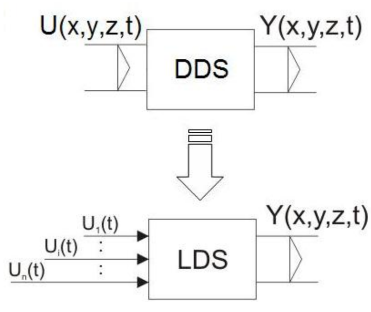


Fig.1 Lumped-input/distributed-output system representation of distributed parameter systems

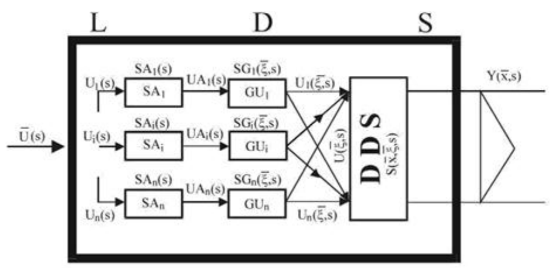


Fig.2 Structure of the LDS: $\{SA_i(s)\}_{i=1,n}$ **– transfer functions of actuating members of lumped variables** $\{SA_i\}_{i=1,n}$**;** $\{SG_i(\bar{\xi},s)\}_{i=1,n}$ **– transfer functions of generators of distributed input variables** $\{GU_i\}_{i=1,n}$**;** $S(\bar{x},\bar{\xi},s)$ **– transfer functions of DDS; where** $\bar{x}$ **,** $\bar{\xi}$ **- are vectors in 3D**

Dynamics of LDS is decomposed to time and space components. In the time dependency, there are for example discrete transfer functions:

$$\left\{SH_i\left(\overline{x}_i, z\right)\right\}_{i=1,n} \tag{1}$$

between i-th input variable and corresponding partial distributed output variable at point $\overline{x}_i = \left(x_i, y_i, z_i\right)$ for i=1, n. In the space dependency there are e. g. reduced transient step responses in steady-state:

$$\left\{\mathcal{H}HR_i\left(\overline{x}, \infty\right)\right\}_{i=1,n} \tag{2}$$

where $\overline{x} = \left(x, y, z\right)$ is vector in 3D.

2. DPS feedback control loop based on LDS

Decomposition of dynamics enables also to decompose the control synthesis to time and space control tasks in distributed parameter control loop, see Fig. 3, where the goal of control is to ensure the steady-state control error to be minimal:

$$\min\left\|E\left(x, \infty\right)\right\| = \min\left\|W\left(\overline{x}, \infty\right) - Y\left(\overline{x}, \infty\right)\right\| \tag{3}$$

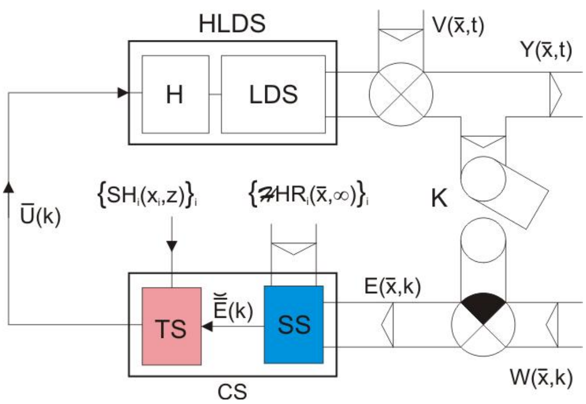


Fig.3 Distributed parameter feedback control loop: HLDS – LDS with zero-order holds $\{H_i\}_i$ on the input, CS – control synthesis, TS – control synthesis in time domain, SS - control synthesis in space domain, K – time/space sampling, $Y\left(\overline{x}, t\right)$ – distributed controlled variable, $W\left(\overline{x}, k\right)$ – control variable, $V\left(\overline{x}, t\right)$ – disturbance variable, $E\left(\overline{x}, k\right)$ – control error

In the block SS, approximation of distributed control error $E\left(\overline{x}, k\right)$, on the set of reduced steady-state distributed step responses $\left\{\mathcal{H}HR_i\left(\overline{x}, \infty\right)\right\}_i$, is solved.

$$\begin{aligned} \min_{E_i}\left\|E\left(\overline{x}, k\right) - \sum_{i=1}^{n} E_i\left(k\right)\mathcal{H}HR_i\left(\overline{x}, \infty\right)\right\| = \\ = \left\|E\left(\overline{x}, k\right) - \sum_{i=1}^{n} \breve{E}_i\left(k\right)\mathcal{H}HR_i\left(\overline{x}, \infty\right)\right\| \end{aligned} \tag{4}$$

As the solution of approximation problem, the control errors vector $\breve{\overline{E}}\left(k\right) = \left\{\breve{E}_i\left(k\right)\right\}_i$ enters into the block TS, where vector of control variables $\overline{U}\left(k\right)$ is generated. The controllers, $\left\{R_i\left(z\right)\right\}_i$ are tuned in single-parameter control loops $\left\{SH_i\left(\overline{x}_i, z\right),\ R_i\left(z\right)\right\}_i$, according to single components of the time part of the controlled distributed parameter system dynamics $\left\{SH_i\left(\overline{x}_i, z\right)\right\}_{i=1,n}$, see Fig. 4.

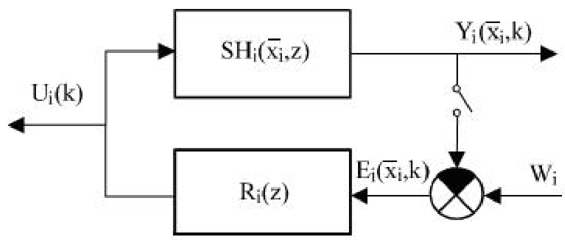


Fig.4 i-th one-parameter control loop

On this principle, the distributed parameter control synthesis at PID, algebraic, state, robust as well as adaptive or intelligent control is decomposed into the time and space tasks, Hulkó (1998 - 2003).

3. Robust control system

In general a mathematical model for the plant dynamics is the basis for analysis and design of control systems. Also for LDS representation of DPS lumped and distributed models are used. However, in practice no mathematical model capable of exactly describing a physical process exists. It is obvious that although no model is able to represent the process perfectly, some of them will do so with greater accuracy than others.

The theory of robust control represents one of the possible approaches to the control system design in the presence of uncertainty. The goal of robust system design is to retain assurance of system performance in spite of model inaccuracies and changes. For the design techniques the following are supposed: formulation of nominal plant model, different plant uncertainty models and requirements both for stability and performance control.

3.1 Sources of uncertainties in the LDS structure and their description

LDS representation of DPS means decomposition of dynamics to space and time components. Both in time and space components uncertainties occur, therefore is very useful to regard it.

In distributed parameter control system according Fig. 5, single-input, single-output control loops in the block TS are tuned as closed feedback control loops using usual methods. In these loops, as models of the controlled system, transfer functions $\left\{SH_i\left(\overline{x}_i, z\right)\right\}_i$ eventually $\left\{S_i\left(\overline{x}_i, s\right)\right\}_i$ in the s-domain are used. These transfer functions describe the dynamics between sequences $\left\{U_i\left(k\right)\right\}_i$ and $\left\{Y_i\left(\overline{x}_i, k\right)\right\}_i$.

Sources of uncertainties are given by:

- procedure of modeling dynamics and possible change of parameters in models (2)
- solution of approximation problem (4), where lumped variables are obtained

In order to consider uncertainties, in this paper will be assumed that the dynamic behaviour of a plant is described not by a single linear time invariant model, but by a family Ψ_i of linear time invariant models. First, we define this family Ψ_i in the frequency domain in following form:

$$\Psi_i = \left\{S_i : \left|\ S_i\left(\overline{x}_i, j\omega\right) - S_i'\left(\overline{x}_i, j\omega\right)\right| \le \overline{L}_{ai}\left(\omega\right)\right\} \tag{5}$$

Here $S_i'(\bar{x}_i, j\omega)$ is the nominal plant. Any member of the family Ψ_i fulfils the conditions:

$$S_i(\bar{x}_i, j\omega) = S_i'(\bar{x}_i, j\omega) + L_{ai}(j\omega) \qquad (6)$$

$$|L_{ai}(j\omega)| \le \bar{L}_{ai}(\omega) \quad , \forall \ S_i \in \Psi_i \qquad (7)$$

where $L_{ai}(j\omega)$ an additive uncertainty and $\bar{L}_{ai}(\omega)$ states a bound on the allowed additive uncertainty. If we wish to work with multiplicative uncertainties, we define relations:

$$L_{mi}(j\omega) = \frac{L_{ai}(j\omega)}{S_i'(\bar{x}_i, j\omega)}, \quad \bar{L}_{mi}(\omega) = \frac{\bar{L}_{ai}(\omega)}{|S_i'(\bar{x}_i, j\omega)|} \qquad (8)$$

3.2 Design of robust controllers

A robust control system for LDS can be designed, for example using the IMC structure (Morari, and Zafiriou, 1989), see Fig. 5. This well-known structure is incorporated into TS block of DPS feedback control system, see Fig. 6.

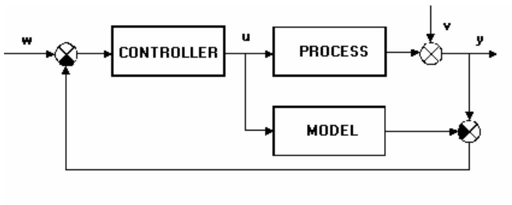


Fig.5 IMC structure for lumped parameter control system

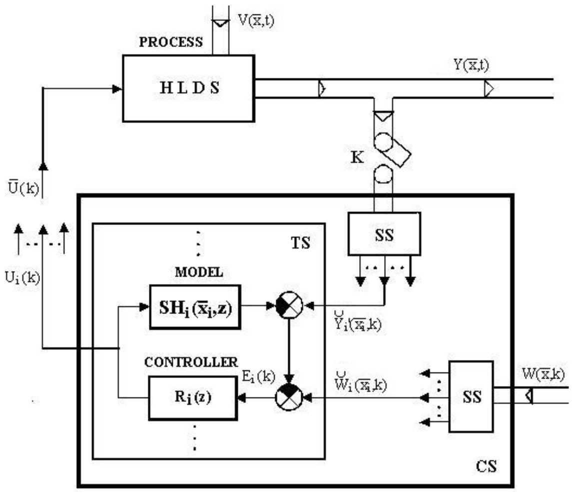


Fig.6 Distributed parameter feedback robust control system

H_2 - optimal controllers $R_i(z)$ are designed by solving the following minimization problem:

$$\min_{R_i(z)} \|e_i(z)\|_2 = \min_{R_i(z)} \left\| \left(1 - SH_i(\bar{x}_i, z) R_i(z)\right) v_i(z) \right\|_2 \qquad (9)$$

subject to the constraint that $R_i(z)$ are stable and causal and where:

$$v_i(z) = V(\bar{x}_i, z) - W(\bar{x}_i, z) \qquad (10)$$

Further factor the models $SH_i(\bar{x}_i, z)$ into an allpass parts $SH_{iA}(\bar{x}_i, z)$ and $SH_{iM}(\bar{x}_i, z)$

$$SH_i(\bar{x}_i, z) = SH_{iA}(\bar{x}_i, z)\, SH_{iM}(\bar{x}_i, z) \qquad (11)$$

and similarly factor the inputs:

$$v_i(z) = v_{iA}\, v_{iM} \qquad (12)$$

Optimal controllers $R_i(z)$ are given by:

$$R_i(z) = z\left(SH_i(\bar{x}_i, z) v_{iM}\right)^{-1} \left\{ z^{-1} SH_{iA}(\bar{x}_i, z)^{-1} v_{iM} \right\}_* \qquad (13)$$

where the operator $\{\ldots\}_*$ means that after a partial fraction expansion of the operand $\{\ldots\}$only the strictly proper and stable (including poles at z=1) are retained.

Controller $R_i(z)$ is augmented by a low-pass filters $F_i(z)$ in the form:

$$F_i(z) = \frac{(1-\alpha_i) z}{z - \alpha_i} \qquad (14)$$

where parameters α_i are chosen with respect to accomplish both robust stability and robust performance condition:

$$\left|F\left(e^{j\omega T}\right)\right| \ \langle \ \left[\left| SH_i(\bar{x}_i, z) R_i\left(e^{j\omega T}\right) \right| \ \bar{L}_{mi}(\omega) \right]^{-1}$$
$$0 \le \omega \le \frac{\pi}{T} \qquad (15)$$

$$\left| R_i(j\omega) \right| \bar{L}_{ai}(\omega) + \left| 1 - S_i(\bar{x}_i, p) R_i(j\omega) \right| G_i(\omega) \ \langle 1$$
$$0 \le \omega \le \frac{\pi}{T} \qquad (16)$$

where $G_i(\omega)$are weighting functions. Then robust controllers in the IMC structure are in the form:

$$\tilde{R}_i(z) = R_i(z) \ F_i(z) \qquad (17)$$

4. Simulation of robust control of temperature field

Robust controllers design procedure outlined above is now applied for the control of temperature field of the mould in the casting process. It is a typical case of DPS, where in the input/output relation is useful to model it as LDS. The benchmark casting plant is built of the steel mould with water-cooled copper inserts (chills), electric heating elements (lumped inputs) and thermocouples, see Fig. 7.

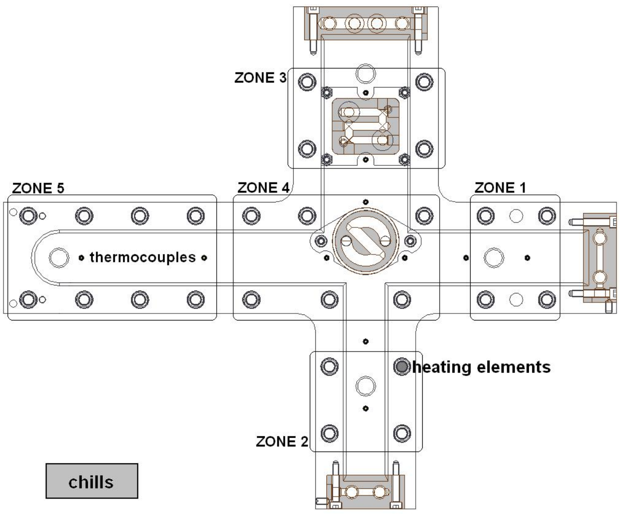


Fig.7 Steel mould of the benchmark casting plant

Distribution of temperatures in the mould in the definition domain $\Omega \in E_3$ is modelled by PDE of parabolic type in the form:

$$\frac{\partial T(\bar{x},t)}{\partial t} - a\nabla^2 T(\bar{x},t) = \sum_i u_i(\bar{x},t) \qquad (18)$$

where $a = \lambda / \rho c$ is temperature conductivity ($m^2.s^{-1}$).

For lumped input variables $\{u_i\}_{i=1,5}$ in the form of step functions, which affect on sub-domains $\{\Omega_i\}_{i=1,5}$ all necessary lumped and distributed responses were computed by FEM in the software environment ProCAST. From point of view uncertainty of models and robust controllers designed, changes of nominal parameters in DPS lumped models (1) were assumed.

Simulation of the control process with five robust controllers was realised in the DPS Blockset for MATLAB & Simulink, see Fig. 8. It is the blockset for distributed parameter systems and their applications in modeling, control and design of dynamical systems given on complex 3D domains of definition.

The block **HLDS** models controlled distributed parameter systems as lumped-input/distributed-output systems with zero-order hold units. The **DPS Control Synthesis** provides feedback to distributed parameter controlled systems in control loops with blocks for **PID, algebraic, state space and robust** control. The block **DPS Input** generates distributed quantities which can be used as distributed control variables or distributed disturbances, etc. **DPS Display** presents distributed quantities with many options including export to AVI files. The block **DPS Space Synthesis** performs space synthesis as an approximation problem. The block **Tutorial** presents methodological framework for formulation and solution of distributed parameter systems of control. The block **Show** contains motivation examples. The block **Demos** contains examples oriented to methodology of modeling and control synthesis. The **DPS Wizard** in step-by-step operation, by means of five model examples on 1D-3D with default parameters, gives a guide for arrangement and setting distributed parameter control loops.

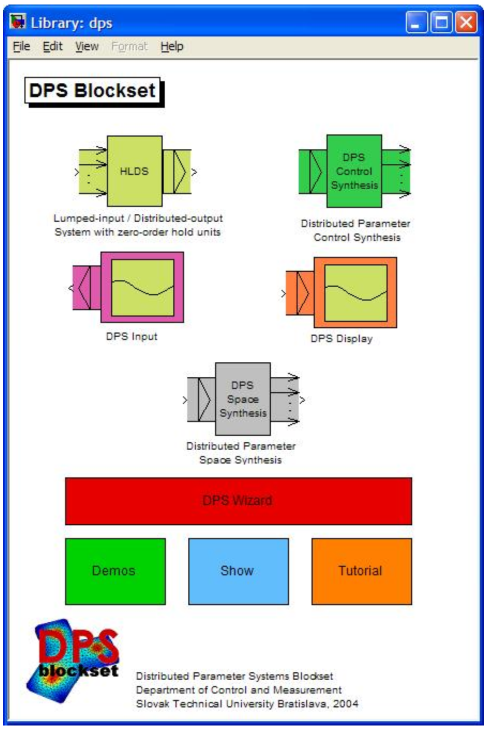


Fig.8 DPS Blockset for MATLAB & Simulink

By means of blocks of the DPS Blockset, DPS feedback robust control system is arranged, see Fig. 9. Parameters of filters $\{\alpha_i\}_{i=1,5}$ in the block DPS Robust synthesis were adjusted in order to assure aperiodic running of the quadratic norm of distributed control error with respect of the robust stability and robust performance conditions. Results of the robust control process are on Fig. 10.

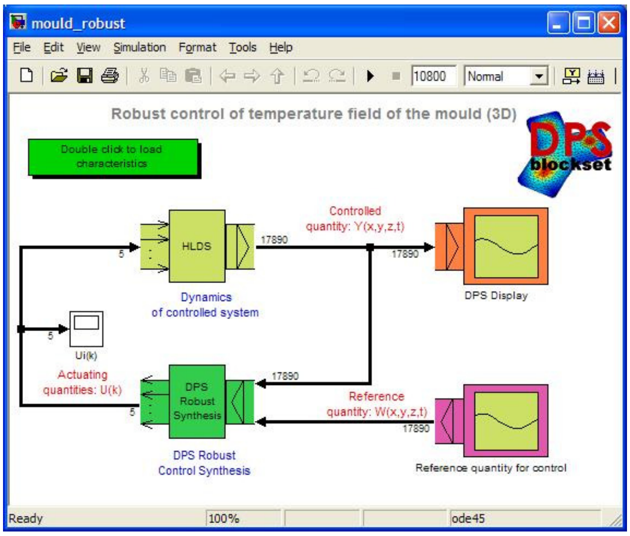
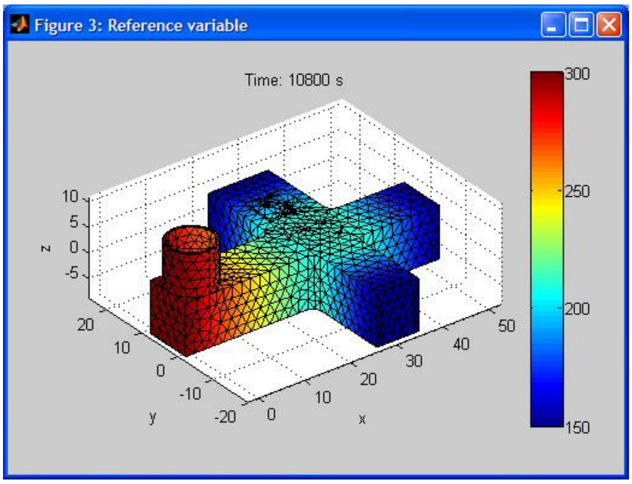
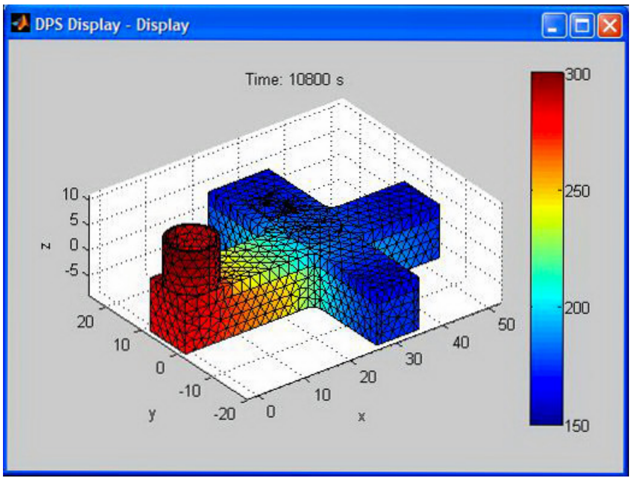


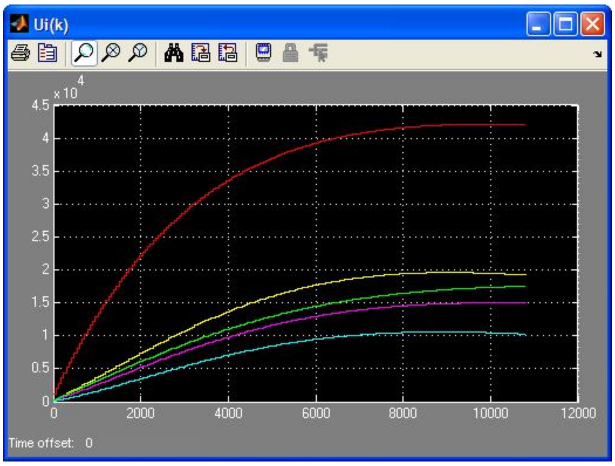
Fig.9 DPS feedback robust control system in the DPS Blockset



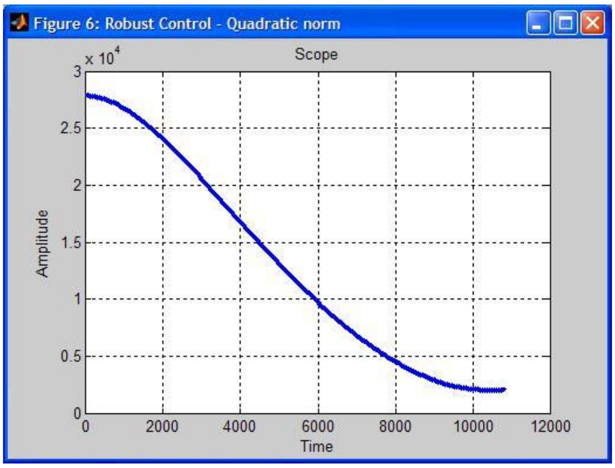
a)



b)



c)



d)

Fig.10 Robust control of temperature field of the casting mould:

a) – distributed reference variable W(x,y,z,∞)

b) – controlled variable Y(x,y,z,∞)

c) – control variables $U_i(k)$

d) – quadratic norm of distributed control error

Conclusion

Development of information technologies supports further wide-ranging distribution of diverse methods and software products for 3D numerical dynamical analysis of real systems as distributed parameter systems in any field of technical practice. Temperature field of the mould in the casting process is a typical case of DPS, where in the input/output relation is useful to model it as LDS.

Methodical approach presented in the paper demonstrates simple possibilities, how to exploit of distributed dynamical characteristics, obtained by numerical FEM analysis of systems on complex definition domains for control synthesis of DPS with respect of an uncertainty of models. The DPS Blockset for MATLAB & Simulink provides block-oriented efficient software for this kind of tasks.

Acknowledgments

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