Adaptive Control of a MIMO Process by Two Feedback Controllers

Petr Dostál, František Gazdoš, Vladimír Bobál, Jiří Vojtěšek

Abstract
The paper deals with continous-time adaptive control of a multi input – multi output nonlinear process. A nonlinear model of the process is approximated by a continuous-time external linear model. The parameters of the CT external linear model are recursively estimated via an external delta model with the same structure as the CT model. The control system configuration with two feedback controllers is considered. Design of controllers is based on operations in the ring of polynomial matrices. Resulting proper controllers ensure asymptotic tracking of step references and step load disturbances attenuation. A control quality is achieved using the exact pole placement method as well as by selectable weight matrices dividing weights among numerators of transfer functions of subcontrollers. The control is tested on a two input – two output nonlinear process represented by a model of two spheric liquid tanks in series.

Keywords: Adaptive control, MIMO system, continuous-time model, delta model, parameter estimation, polynomial method.

Introduction

A wide range of technological processes requires to control more output signals independently. In order to achieve this, it is necessary to have at least as many independent input signals as output signals to be controlled. These processes, classified as MIMO (multi input-multi output) systems, are usually nonlinear. This fact may cause difficulties when controlling such processes using conventional controllers with fixed parameters. One possible method to cope with this problem is using adaptive strategies based on an appropriate choice of an external linear model (ELM) with recursively estimated parameters. These parameters are consequently used for parallel updating of controller's parameters.

The control itself can be either continuous-time or discrete. While for design of a continuous-time controller, it is necessary to know a continuous-time ELM and its parameters, a discrete-time controller requires knowledge of a discrete ELM. Experiences of many authors in the field of control of non-linear technological processes indicate that the continuous-time (CT) approach gives better results when controlling processes with strong nonlinearities.

Two basic approaches can be used for identification of the continuous-time ELM. The first method is based on filtration of input and output signals where the filtered variables have the same properties (in the *s*-domain) as their non-filtered counterparts, e.g. [1]. Derivatives of filtered signals that are necessary for the parameters estimate of the CT ELM are obtained from differential filters. This method has, however, some drawbacks – the necessity to solve additional differential equations representing the filters and estimate time constants of these filters.

The second strategy uses an external δ-model of the controlled process with the same structure as a CT model. The basics of δ-models have been described in e.g. [2], [3]. Here, parameters of δ-models can directly be estimated from sampled signals without the necessity to filter them. Moreover, it can be easily proved that these parameters converge to parameters of CT models for a sufficiently small sampling period (compared to the dynamics of the controlled process). A complete description and experimental verification can be found in e.g. [4]. The control results obtained using both mentioted strategies were compared in [5].

This contribution deals with adaptive control of a nonlinear MIMO process. The parameters of the CT ELM of the process are obtained via corresponding delta model parameter estimation. The control configuration with two feedback controllers is used according to [6]. Input signals for the control system are step references and step load disturbances. Resulting controllers are derived by the polynomial method [7], [8]. The approach is tested on a nonlinear TITO process represented by a nonlinear model of two spheric liquid tanks in series.

CT External Linear Model

In the time domain, the generalized continuous-time ELM is specified by the vector differential equation

$$\boldsymbol{A}(\sigma)\,\boldsymbol{y}(t) = \boldsymbol{B}(\sigma)\,\boldsymbol{u}(t) \tag{1}$$

where $\sigma = d/dt$ is the derivative operator, $\boldsymbol{y} \in \Re^r$ stands for the controlled output vector, $\boldsymbol{u} \in \Re^m$ is the control input vector and $\boldsymbol{A}$, $\boldsymbol{B}$ are polynomial matrices in σ. Using the Laplace transform under zero initial conditions, the model is described in the *s*-domain as

$$\boldsymbol{Y}(s) = \boldsymbol{G}(s)\,\boldsymbol{U}(s)\,. \tag{2}$$

Here, the transfer function of the controlled system is assumed in the form of the left coprime polynomial matrix fraction

$$\boldsymbol{G}(s) = \boldsymbol{A}^{-1}(s)\boldsymbol{B}(s) \tag{3}$$

where $\boldsymbol{A}(s) \in \Re^{rr}[s]$ and $\boldsymbol{B}(s) \in \Re^{rm}[s]$ are polynomial matrices. Further, consider strictly proper $\boldsymbol{G}(s)$, and, with regard to some following operations, assume the highest power of s on the diagonal of the matrix $\boldsymbol{A}$ in each row. Moreover, the polynomials on the diagonal are assumed to be monic polynomials (with the unit coefficient by the highest power of s).

External Delta Model

Establish the δ-operator defined by

$$\delta = \frac{q-1}{T_0} \tag{4}$$

where q is the forward shift operator and T_0 is the sampling interval. When the sampling interval is shortened, the δ-operator approaches the derivative operator σ so that

$$\lim_{T_0 \to 0} \delta = \sigma \tag{5}$$

and, the δ-model

$$\boldsymbol{A}'(\delta)\boldsymbol{y}(t') = \boldsymbol{B}'(\delta)\boldsymbol{u}(t') \tag{6}$$

approaches the continuous-time model (1). Here, t' is the discrete time, and, $\boldsymbol{A}'$ and $\boldsymbol{B}'$ are matrices with identical structure as $\boldsymbol{A}$ and $\boldsymbol{B}$ in the form

$$\boldsymbol{A}'(\delta) = \begin{pmatrix} a'_{11}(\delta) & .. & a'_{1i}(\delta) & .. & a'_{1r}(\delta) \\ \vdots & & \vdots & & \vdots \\ a'_{i1}(\delta) & .. & a'_{ii}(\delta) & .. & a'_{ir}(\delta) \\ \vdots & & \vdots & & \vdots \\ a'_{r1}(\delta) & .. & a'_{ri}(\delta) & .. & a'_{rr}(\delta) \end{pmatrix} \tag{7}$$

$$\boldsymbol{B}'(\delta) = \begin{pmatrix} b'_{11}(\delta) & .. & b'_{1m}(\delta) & .. & b'_{1m}(\delta) \\ \vdots & & \vdots & & \vdots \\ b'_{i1}(\delta) & .. & b'_{ii}(\delta) & .. & b'_{im}(\delta) \\ \vdots & & \vdots & & \vdots \\ b'_{r1}(\delta) & .. & b'_{ri}(\delta) & .. & b'_{rm}(\delta) \end{pmatrix} \tag{8}$$

with polynomials

$$a'_{ij}(\delta) = a'_{n_{ij},ij}\delta^{n_{ij}} + a'_{n_{ij}-1,ij}\delta^{n_{ij}-1} + \ldots + a'_{1,ij}\delta + a'_{0,ij} \tag{9}$$

$$b'_{ik}(\delta) = b'_{m_{ik},ik}\delta^{m_{ik}} + \ldots + b'_{1,ik}\delta + b'_{0,ik} \tag{10}$$

where

$a'_{n_{ii},ii} = 1$, $n_{ii} > n_{ij}$ for $j \neq i$ and $n_{ii} > m_{ik}$ for all i, j = 1, … , r and k = 1, … , m.

Substituting $t' = k_0 - n_{ii}$ where $k_0 \geq n_{ii}$, the equation describing i-th row of (6) can be derived as

$$\sum_{j=0}^{n_{i1}} a'_{j,i1}\delta^j y_1(k_0 - n_{ii}) + \sum_{j=0}^{n_{ii}} a'_{j,ii}\delta^j y_i(k_0 - n_{ii}) + \ldots$$
$$\ldots + \sum_{j=0}^{n_{ir}} a'_{j,ir}\delta^j y_r(k_0 - n_{ii}) = \tag{11}$$
$$= \sum_{j=0}^{m_{i1}} b'_{j,i1}\delta^j u_1(k_0 - n_{ii}) + \sum_{j=0}^{m_{im}} b'_{j,im}\delta^j u_m(k_0 - n_{ii})$$

where the terms in (11) are

$$\delta^{n_{ij}} y_i(k_0 - n_{ii}) = \sum_{p=0}^{n_{ij}} \frac{(-1)^p}{T_0^{n_{ij}}}\binom{n_{ij}}{p} y_i(k_0 - n_{ii} + n_{ij} - p)$$
$$\delta^{m_{ik}} u_k(k_0 - n_{ii}) = \sum_{p=0}^{m_{ik}} \frac{(-1)^p}{T_0^{m_{ik}}}\binom{m_{ik}}{p} u_k(k_0 - n_{ii} + m_{ik} - p). \tag{12}$$

Delta Model Parameter Estimation

Obviously, an actual value of the controlled output $y_i(k_0)$ in the i-th row is only in the term $\delta^{n_{ii}} y_i(k_0 - n_{ii})$ (for j = i and p = 0 in (12)). Now, denoting

$$\varphi^j_{i,y_i} = \delta^j y_i(k_0 - n_{ii}), \quad \varphi^j_{i,u_k} = \delta^j u_k(k_0 - n_{ii}) \tag{14}$$

and, introducing the regression vector

$$\boldsymbol{\Phi}^T_{\delta i} = \Big(-\varphi^0_{i,y_1} \ldots -\varphi^{n_{i1}}_{i,y_1}, \ldots, -\varphi^0_{i,y_i} \ldots -\varphi^{n_{ii}-1}_{i,y_i}, \ldots,$$
$$-\varphi^0_{i,y_r} \ldots -\varphi^{n_{ir}}_{i,y_r}, \varphi^0_{i,u_1} \ldots \varphi^{m_{i1}}_{i,u_1}, \ldots, \varphi^0_{i,u_m} \ldots \varphi^{m_{im}}_{i,u_m}\Big) \tag{15}$$

then, the vector of parameters in the i-th row of $\boldsymbol{A}'$

$$\boldsymbol{\Theta}^T_i = \Big(a'_{0,i1} \ldots a'_{n_{i1},i1}, \ldots, a'_{0,ii} \ldots a'_{n_{ii}-1,ii}, \ldots,$$
$$a'_{0,ir} \ldots a'_{n_{ir},ir}, b'_{0,i1} \ldots b'_{m_{i1},i1}, \ldots, b'_{0,im} \ldots b'_{m_{im},im}\Big) \tag{16}$$

can be recursively estimated from the regression (ARX) model

$$\varphi^{n_{ii}}_{i,y_i} = \boldsymbol{\Theta}^T_i \boldsymbol{\Phi}_{\delta i} + \varepsilon_i(k_0). \tag{17}$$

or, in detail, from the equation

$$\delta^{n_{ii}} y_i(k_0 - n_{ii}) = -\sum_{j=0}^{n_{i1}} a'_{j,i1}\delta^j y_1(k_0 - n_{ii}) - \ldots$$
$$-\sum_{j=0}^{n_{ii}-1} a'_{j,ii}\delta^j y_i(k_0 - n_{ii}) - \ldots - \sum_{j=0}^{n_{ir}} a'_{j,ir}\delta^j y_r(k_0 - n_{ii}) +$$
$$+\sum_{j=0}^{m_{i1}} b'_{j,i1}\delta^j u_1(k_0 - n_{ii}) + \ldots \tag{18}$$
$$+\sum_{j=0}^{m_{im}} b'_{j,im}\delta^j u_m(k_0 - n_{ii}) + \varepsilon_i(k_0).$$

Controller Design

The control system with two feedback controllers is depicted in Fig. 1. Here, $\boldsymbol{G}$ represents the continuous-time ELM, $\boldsymbol{G}_Q$ and $\boldsymbol{G}_R$ are controllers. Further, $\boldsymbol{w} \in \Re^r$ is the vector of references and $\boldsymbol{v} \in \Re^m$ is the vector of load disturbances.

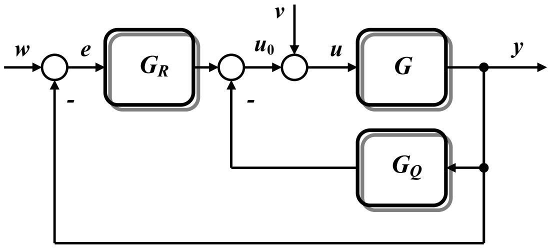

Fig.1 Control system configuration.

Generally, their transforms can be expressed as

$$w(s)=F_w^{-1}(s)h_w(s)\,,\;\; v(s)=F_v^{-1}(s)h_v(s) \tag{19}$$

Considering all elements of both input signals as step function, matrices F_w and F_v in (19) take forms

$$F_w(s)=F_v(s)=s\,I \tag{20}$$

and vectors (19) can be rewritten to as

$$W(s)=\begin{pmatrix}\frac{w_{10}}{s} & \frac{w_{20}}{s} & \ldots & \frac{w_{r0}}{s}\end{pmatrix}^T \tag{21}$$

$$V(s)=\begin{pmatrix}\frac{v_{10}}{s} & \frac{v_{20}}{s} & \ldots & \frac{v_{m0}}{s}\end{pmatrix}^T \tag{22}$$

where w_{i0} , v_{j0} are constants.

The transfer functions of controllers are assumed in the form of right coprime polynomial matrix fractions

$$G_Q(s)=Q_1(s)P_1^{-1}(s)\;,\; G_R(s)=R_1(s)P_1^{-1}(s) \tag{23}$$

where

$Q_1(s)\in\Re^{mr}[s]$, $R_1(s)\in\Re^{mr}[s]$ and $P_1(s)\in\Re^{rr}[s]$.

The goal is to find such proper controllers that ensure the control system stability, asymptotic tracking of step references and step load disturbance attenuation. The procedure for deriving admissible controllers can be performed as follows:

Using descriptions of basic signals in the control system

$$y(s)=A^{-1}B\,u(s)=A^{-1}B\left[u_0(s)+v(s)\right] \tag{24}$$

$$u_0(s)=R_1P_1^{-1}\left[w(s)-y(s)\right]-Q_1P_1^{-1}y(s) \tag{25}$$

the output and tracking error vectors can be derived as

$$y(s)=P_1D^{-1}\left[B\,R_1P_1^{-1}w(s)+B\,v(s)\right] \tag{26}$$

$$e(s)=P_1D^{-1}\left[(AP_1+BQ_1)P_1^{-1}w(s)-B\,v(s)\right] \tag{27}$$

vhere

$$D=AP_1+B\,(R_1+Q_1)\,. \tag{28}$$

Now, feedback controllers given by a solution of the matrix Diophantine equation

$$AP_1+BT=D \tag{29}$$

with a stable polynomial matrix $D\in\Re^{rr}[s]$ on the right side ensure the control system stability. Here, the matrix T has been established as

$$T=R_1+Q_1\,. \tag{30}$$

The step load disturbances will be rejected for the matrix P_1 in (27) divisible by denominators *s* in (22).

This condition is fulfilled for P_1 in the form

$$P_1(s)=s\,\tilde{P}_1(s)\,. \tag{31}$$

Asymptotic tracking of step references is ensured for the term AP_1+BQ_1 divisible by s in denominators of (21).

Considering (27) and (31), this divisibility is fulfilled for Q_1 taking the form

$$Q_1(s)=s\tilde{Q}_1(s)\,. \tag{32}$$

Taking into account (31), (32) and (29), the controller polynomial matrices are given by a solution of the matrix Diophantine equation

$$A(s)\,s\,\tilde{P}_1(s)+B(s)\,T(s)=D(s) \tag{33}$$

where

$$T(s)=R_1(s)+s\,\tilde{Q}_1(s)\,. \tag{34}$$

Evidently, the degrees of matrices in (34) are given as

$$\deg R_1=\deg T\;,\;\; \deg\tilde{Q}_1=\deg T-1\,. \tag{35}$$

Considering expansions of matrices T **,** R_1 and $\tilde{Q}_1$ as

$$T(s)=\sum_{j=0}^{\deg T}s^jT_j \tag{36}$$

$$R_1(s)=\sum_{j=0}^{\deg T}s^jR_{1j} \tag{37}$$

$$\tilde{Q}_1(s)=\sum_{j=1}^{\deg T}s^{j-1}\tilde{Q}_{1j} \tag{38}$$

where T_j , R_{1j} and $\tilde{Q}_{1j}$ are matrices of coefficients, a solution of (33) leads to a simple term of T given by

$$B_0T_0=D_0 \tag{39}$$

and, subsequently, to

$$R_{10}=T_0\,. \tag{40}$$

It is well known that a solution of a single polynomial matrix equation provides only two unknown polynomial matrices. Hence, selectable coefficient matrices $\Gamma_j\in\Re^{mm}$ can be introduced that distribute weights among R_1 and $\tilde{Q}_1$ parameters. Denoting expansions of matrices R_1 and $\tilde{Q}_1$ as

$$R_{1j}\,,\tilde{Q}_{1j}\;,\; j=1,\ldots,\deg T \tag{41}$$

then, their elements can be calculated from equations

$$R_{1j}=\Gamma_jT_j\,,\;\; \tilde{Q}_{1j}=\left(I-\Gamma_j\right)T_j \tag{42}$$

for $j=1,\ldots,\deg T$.

Remark: If $\boldsymbol{\Gamma}_j = \boldsymbol{I}$ for all j, the control system in Fig. 1 simplifies to the 1DOF control configuration. If $\boldsymbol{\Gamma}_j = \boldsymbol{0}$ for all j, and, both references and load disturbances are step functions, the control system corresponds to the 2DOF control configuration.

From the practical point of view, it is effective to choose Γ_j as diagonal matrices

$$\boldsymbol{\Gamma}_j = \begin{pmatrix} \gamma_{j1} & .. & .. & 0 \\ : & \gamma_{j2} & & \\ : & & .. & \\ 0 & & & \gamma_{jm} \end{pmatrix} \tag{43}$$

for all j.

Now, taking into account (31) and (32), transfer functions of controllers can be rewritten to the form

$$\boldsymbol{G}_Q(s) = \tilde{\boldsymbol{Q}}_1(s)\left(\tilde{\boldsymbol{P}}_1(s)\right)^{-1} \tag{44}$$

$$\boldsymbol{G}_R(s) = \boldsymbol{R}_1(s)\left(s\tilde{\boldsymbol{P}}_1(s)\right)^{-1}. \tag{45}$$

Note that degrees of polynomial matrices in (33) must be determined in accordance with the requirement on properness of controller transfer functions (44) and (45).

Example

Consider two spheric liquid tanks in series as depicted in Fig. 2.

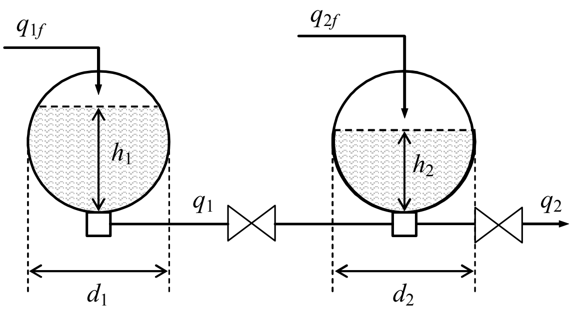

Fig.2 Two spheric liquid tanks in series.

Using standard simplifications, the model of the plant can be described by two nonlinear differential equations

$$\pi h_1(d_1 - h_1)\frac{dh_1}{dt} + q_1 = q_{1f} \tag{46}$$

$$\pi h_2(d_2 - h_2)\frac{dh_2}{dt} - q_1 + q_2 = q_{2f} \tag{47}$$

where d_j are diameters of tanks, h_j are liquid levels, q_j are stream flowrates and q_{jf} are their inlet values, (for j = 1, 2). The stream volumetric flowrates depend upon levels in the tanks as

$$q_1 = k_1\sqrt{|h_1 - h_2|} \tag{48}$$

$$q_2 = k_2\sqrt{h_2} \quad (\text{if } h_1 - h_2 < 0 \text{ then } q_1 = -q_1) \tag{49}$$

where k_1, k_2 are constants.

Initial conditions for (46), (47) are steady state liquid levels $h_1(0) = h_1^s$, $h_2(0) = h_2^s$. The model parameters and values of variables at the operating point used in simulations are:

$k_1 = 0.85\,\mathrm{m}^{2.5}/\min$, $k_2 = 0.5\,\mathrm{m}^{2.5}/\min$, $d_1 = d_2 = 2\,\mathrm{m}$,

$h_1^s = 1.5\,\mathrm{m}$, $h_2^s = 1.3\,\mathrm{m}$, $q_{1f}^s = 0.38\,\mathrm{m}^3/\min$

and $q_{2f}^s = 0.19\,\mathrm{m}^3/\min$.

Both the control and controlled variables are considered as deviations from their values at the operating point

$$u_1(t) = q_{1f}(t) - q_{1f}^s, \quad u_2(t) = q_{2f}(t) - q_{2f}^s \tag{50}$$

$$y_1(t) = h_1(t) - h_1^s, \quad y_2(t) = h_2(t) - h_2^s. \tag{51}$$

Polynomial matrices of the CT external linear model in the form of LPMF have been chosen as

$$\boldsymbol{A}(s) = \begin{pmatrix} s + a_{01} & a_{02} \\ a_{03} & s + a_{04} \end{pmatrix}, \quad \boldsymbol{B}(s) = \begin{pmatrix} b_{01} & 0 \\ 0 & b_{04} \end{pmatrix} \tag{52}$$

and their parameters were estimated using a δ-model with corresponding matrices

$$\boldsymbol{A}'(\delta) = \begin{pmatrix} \delta + a'_{01} & a'_{02} \\ a'_{03} & \delta + a'_{04} \end{pmatrix}, \boldsymbol{B}'(\delta) = \begin{pmatrix} b'_{01} & 0 \\ 0 & b'_{04} \end{pmatrix}. \tag{53}$$

Further, two parallel identification procedures according to (18) were used for regression equations

$$\begin{aligned} \delta y_1(k_0 - 1) = {} & b'_{01}u_1(k_0 - 1) - a'_{01}y_1(k_0 - 1) - \\ & - a'_{03}y_2(k_0 - 1) + \varepsilon_1(k_0) \end{aligned} \tag{54}$$

$$\begin{aligned} \delta y_2(k_0 - 1) = {} & b'_{04}u_1(k_0 - 1) - a'_{02}y_1(k_0 - 1) - \\ & - a'_{04}y_2(k_0 - 1) + \varepsilon_2(k_0) \end{aligned} \tag{55}$$

where

$$\delta y_i(k_0 - 1) = \frac{y_i(k_0) - y_i(k_0 - 1)}{T_0}, \quad i = 1,2. \tag{56}$$

Here, the recursive identification method with exponential and directional forgetting according to [9].

With regard to requirement of the controller properness, matrices $\boldsymbol{P}_1$ and $\boldsymbol{T}$ have been chosen in the form

$$\boldsymbol{P}_1(s) = s\,\tilde{\boldsymbol{P}}_1(s) = \begin{pmatrix} s\,p_{01} & s\,p_{02} \\ s\,p_{03} & s\,p_{04} \end{pmatrix} \tag{57}$$

$$\boldsymbol{T}(s) = \begin{pmatrix} t_{11}s + t_{01} & t_{12}s + t_{02} \\ t_{13}s + t_{03} & t_{14}s + t_{04} \end{pmatrix} \tag{58}$$

and, the diagonal matrix on the right side of (33) as

$$\boldsymbol{D}(s) = \begin{pmatrix} (s + \alpha)^2 & 0 \\ 0 & (s + \alpha)^2 \end{pmatrix} \tag{59}$$

Then, solving (33), the coefficients in (57) and (58) can be derived as

$$p_{02} = p_{03} = 0, \quad p_{01} = p_{04} = 1,$$

$$t_{01} = \frac{\alpha^2}{b'_{01}}, \quad t_{11} = \frac{1}{b'_{01}}(2\alpha - a'_{01}), \; t_{02} = t_{03} = 0, \tag{60}$$

$$t_{12} = -\frac{a'_{02}}{b'_{01}}, \; t_{13} = -\frac{a'_{03}}{b'_{04}}, \; t_{04} = \frac{\alpha^2}{b'_{04}}, \quad t_{14} = \frac{1}{b'_{04}}(2\alpha - a'_{04}).$$

Choosing the matrix (43) as

$$\boldsymbol{\Gamma}_1 = \begin{pmatrix} \gamma_{11} & 0 \\ 0 & \gamma_{12} \end{pmatrix} \tag{61}$$

and, solving (42), transfer functions of controllers take forms

$$\boldsymbol{G}_Q(s) = \begin{pmatrix} (1-\gamma_{11})t_{11} & (1-\gamma_{11})t_{12} \\ (1-\gamma_{12})t_{13} & (1-\gamma_{12})t_{14} \end{pmatrix} \tag{62}$$

$$\boldsymbol{G}_R(s) = \begin{pmatrix} \gamma_{11}t_{11} + \dfrac{t_{01}}{s} & \gamma_{11}t_{12} \\ \gamma_{12}t_{13} & \gamma_{12}t_{14} + \dfrac{t_{04}}{s} \end{pmatrix}. \tag{63}$$

Simulation Results

The recursive estimation of δ-model parameters was performed with the sampling interval T_0 = 0.05 min. in all simulation experiments. For the start, P controllers with a small gain were used. In most simulations, the quadruple closed-loop pole α = 0.4 has been chosen.

In the first case, the controlled output and control input time responses were simulated to step references $w_1 = 0.1$ and $w_2 = 0.05$ for $t \geq 0$. The responses in Figs. 3, 4 and 5 clearly illustrate the effect of parameters γ upon control properties. Their hihger values accelerate the control, however, they lead to overshoots in control output responses and to higher values of the control input.

Next simulations show the effect of the parameter α versus parameters γ on the controlled output responses. While α affects both controlled outputs, by a suitable γ selection only single controlled output can be influenced, as shown in Figs. 6 – 8.

The controlled output responses for step changes of references are shown in Fig. 9. Presented results demonstrate that by a suitable selection of parameters α and γ, smooth control responses of a good quality without overshoots and oscillations can be obtained.

The simulation results in Fig. 10 show the controlled output responses to step references and load disturbances $v(t) = \pm 0.2$. Here, the controller parameters were estimated only in the first (tracking) interval t < 30. During interval $t \geq$ 30, fixed parameters were used.

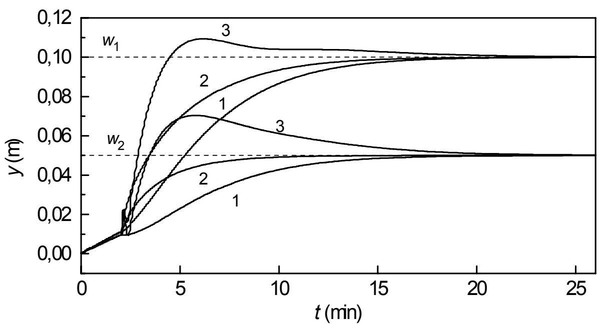

Fig.3 Controlled output responses

α = 0.4, $\gamma_{11} = \gamma_{12}$ = 0, 2 (1), $\gamma_{11} = \gamma_{12}$ = 0.4 (2), $\gamma_{11} = \gamma_{12}$ = 1 (3).

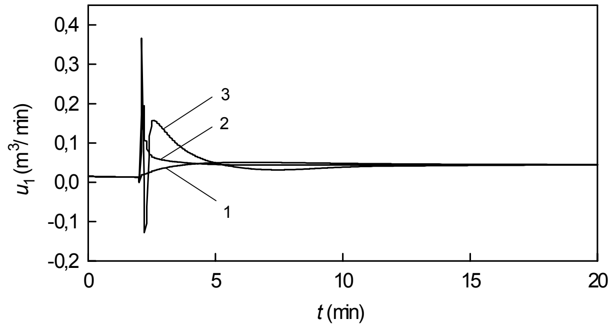

Fig.4 Control input responses

α = 0.4, $\gamma_{11} = \gamma_{12}$ = 0 (1), $\gamma_{11} = \gamma_{12}$ = 0.4 (2), $\gamma_{11} = \gamma_{12}$ = 1 (3).

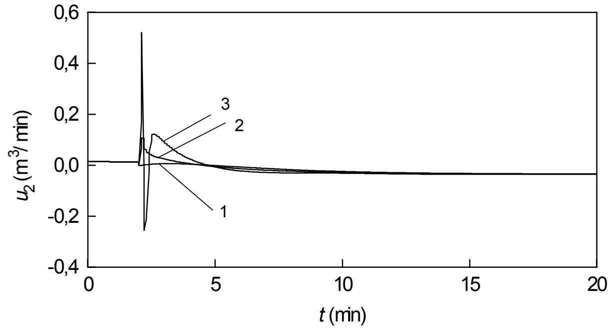

Fig.5 Control input responses

α = 0.4, $\gamma_{11} = \gamma_{12}$ = 0 (1), $\gamma_{11} = \gamma_{12}$ = 0.4 (2), $\gamma_{11} = \gamma_{12}$ = 1 (3).

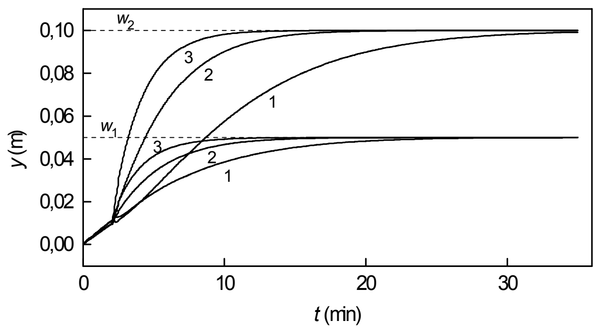

Fig.6 Controlled output responses

$\gamma_{11} = \gamma_{12}$ = 0.2, α = 0.2 (1), α = 0.4 (2), α = 0.6 (3).

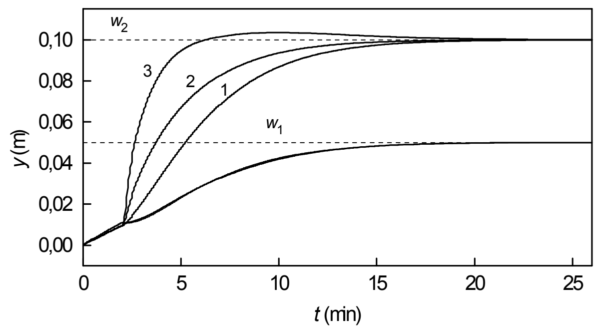

Fig.7 Controlled output responses

α = 0.4, γ_{12} = 0, γ_{11} = 0 (1), γ_{11} = 0.2 (2), γ_{11} = 0.5 (3).

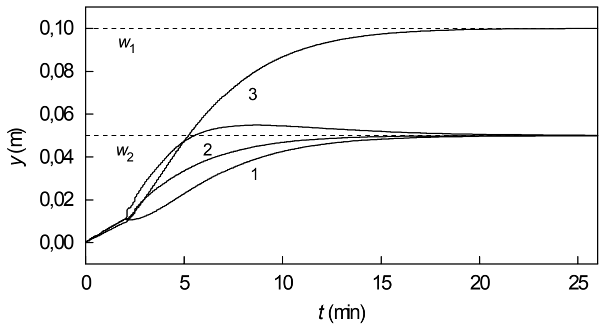

Fig.8 Controlled output responses

α = 0.4, γ_{11} = 0, γ_{12} = 0, 2 (1), γ_{12} = 0.4 (2), γ_{12} = 1 (3).

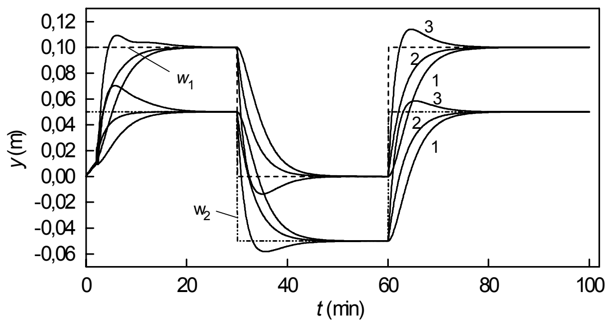

Fig.9 Controlled output responses

α = 0.4, γ_{11} = γ_{12} = 0, 2 (1), γ_{11} = γ_{12} = 0.4 (2), γ_{11} = γ_{12} = 1 (3).

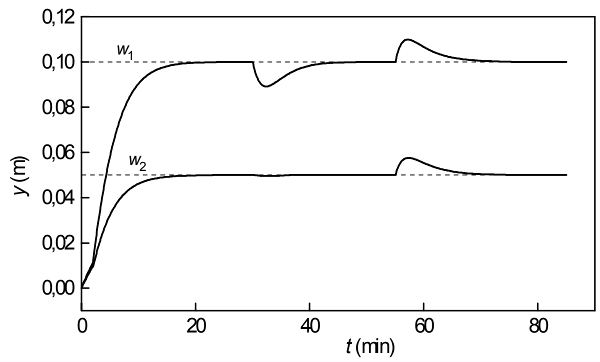

Fig.10 Controlled output responses to disturbances

α = 0.4, γ_{11} = γ_{12} = 0 (1), γ_{11} = γ_{12} = 0.5 (2).

Conclusions

In this paper, one approach to the continuous-time control of nonlinear multi input-multi output processes was proposed. The presented strategy enables to create an effective control algorithm. This algorithm is based on an alternative continuous-time external linear model in the form of the left polynomial matrix fraction with parameters recursively estimated using a corresponding δ-model. The control configuration with two feedback controllers is used. Both resulting continuous-time controllers are given by a solution of polynomial matrix Diophantine equations. Parameters of the controller are periodically readjusted according to recursively estimated parameters of the δ-model. The controller parameters can be tuned by selectable poles of the closed-loop as well as by parameters distributing weights among numerators of subcontroller transfer funktions. The presented method has been tested by computer simulation on the nonlinear model of two spheric tanks in series. The results demonstrate the applicability of the presented control strategy. It can be deduced that the described adaptive strategy is also suitable for other technological processes.

Acknowledgments

This work was supported in part by the Ministry of Education of the Czech Republic under grant MSM 7088352101 and by the Grant Agency of the Czech Republic under grants No. 102/05/0271 and No. 102/06/1132.

References

[1] WAHLBERG, B.: The effects of rapid sampling in system identification, Automatica, 26, 167-170, 1990.

[2] MIDDLETON, R.H., GOODWIN, G.C.: Digital Control and Estimation - A Unified Approach. Prentice Hall, Englewood Cliffs, 1990.

[3] MUKHOPADHYAY, S., PATRA, A.G., RAO, G.P.: New class of discrete-time models for continuos-time systems. International Journal of Control, 55, 1992, 1161-1187.

[4] STERICKER, D.L., SINHA, N.K.: Identification of continuous-time systems from samples of input-output data using the δ-operator. Control-Theory and Advanced Technology, 9, 1993, 113-125.

[5] DOSTÁL, P., BOBÁL, V., GAZDOŠ, F.: Adaptive control of nonlinear processes: Continuous-time versus delta model parameter estimation. In: Proc. IFAC Workshop on Adaptation and Learning in Control and Signal Processing ALCOSP 04, Yokohama, Japan, 2004, 273-278.

[6] ORTEGA, R., KELLY, R.: PID self-tuners: Some theoretical and practical aspects. IEEE Trans. Ind. Electron., Vol. IE-31, 1984, 332-338.

[7] KUČERA, V.: Diophantine equations in control – A survey. Automatica, 29, 1993, 1361-1375.

[8] GRIMBLE, M.J.: Robust industrial control. Optimal design approach for polynomial systems, Prentice Hall, Englewood Cliffs, 1993.

[9] KULHAVÝ, R., KÁRNÝ, M.: Tracking of slowly varying parameters by directional forgetting. In: Proc. 9th IFAC World Congress, Vol. X, Budapest, 78-83, 1984.

prof. Ing. Petr Dostál, CSc.
Ing. František Gazdoš, Ph.D.
prof. Ing. Vladimír Bobál, CSc.
Ing. Jiří Vojtěšek

Tomas Bata University in Zlin
Faculty of Applied Infortmatics
Department of Process Control
Nad Stráněmi 4511
760 05 Zlín
e-mail: dostalp@fai.utb.cz