Adaptive dual control of three tank system

Vladimír Bobál, Petr Chalupa, Petr Dostál

Abstract
The general task of optimal adaptive control with recursive identification (self-tuning control) is very complicated problem. This problem is solved usually by the separation of identification and control – the Certainty Equivalency (CE) Principle. The aim of this paper is to present the solution of this problem using the Dual Adaptive Control (Bicriterial Approach). The main idea of this approach involves two cost functions: (1) the system output should track cautiously the desired reference signal; (2) the control signal should excite the controlled process sufficiently for accelerating the parameter estimates. This approach was verified by a real-time control of nonlinear time varying laboratory model – DTS200 Three Tank System.

Keywords: Self-tuning algorithms, Dual control, Autoregressive models, Recursive least squares method, Nonlinear systems, Real-time systems

Introduction

One approach to adaptive control is based on the recursive estimation of the unknown system characteristics, their gradual specification and thus monitoring possible changes. Using this knowledge, appropriate methods can be employed to design the optimal controller. This kind of controller, which identifies unknown processes and then synthesizes control (adaptive control with recursive identification) is referred in the literature as a *self-tuning controller* - STC.

It is clear that to reach these goals the identification of the static and dynamic characteristics of a controlled process plays an important role together with the optimal control strategy itself. It is known from parameter estimation theory that the determination of parameters is always burdened by a degree of uncertainty - error. This uncertainty not only depends on the number of identified steps (i.e. the amount of sample data) and on the choice of structure for the mathematical model of the controlled process, but is also dependent on the behaviour of the controller output, the sampling period and the choice of filter for the controller and process outputs. This means that every realized change in the controller output except the required control effect also excites the controlled system and thus creates the condition for its identification; in other words, for the best identification of the controlled process, it is necessary to impose certain conditions on the course of controller inputs.

The general task of optimal adaptive control with recursive identification is, therefore, extremely complicated. The controller output signal of optimal adaptive system should have two main properties:

- it has ensure that the process output follows the reference signal value and respond to its changes,
- it has to excite sufficiently the controlled process for its quality identification.

These properties are introduced in the literature as *dual properties* (or *dual features*) and adaptive control system giving these two properties is indicated as *adaptive dual control systems*.

The exact solution to the optimal dual adaptive control was presented by Feldbaum [1], [2] using the dynamic programming. Unfortunately, due to the complexity of calculations it involves, exact dual optimal control is too demanding to be of use in most situations.

It has, therefore, been necessary to simplify the solution to this problem using experimental experience and intuition. This solution is based on constrained separation of identification and control - the *Certainty Equivalence* (CE) *Principle*. The principle of CE consists in the fact that the model uncertainty is not considered. For the controller design the parameter estimates of the process model, which are obtained by the recursive identification, are used. It is assumed at the same time that values of these estimates correspond to their real values. It is obvious that adaptive control systems based on CE approach are not always optimal. For that purpose, several simplified approaches to design of adaptive control systems have been developed. These simplifications could be divided into two main groups based on: (1) *approximations* of the dual problem known as *implicit dual control methods*; (2) *reformulation* of the problem known as *explicit dual control methods* [3], [4], [5].

One of the most efficient approaches is given by the bicriterial synthesis method for dual adaptive controllers. The main idea of the bicriterial approach consists in the introduction of two cost functions that correspond to the two goals of dual control: (1) to track the plant output to the desired reference signal and (2) to introduce the excitation up the parameter estimation. This bicriterial approach has been designed essentially by Filatov and Unbehauen [6]. In this paper the bicriterial approach is used for adaptive dual control of the DTS200 laboratory model.

1. Structure of adaptive dual systems

The main difference between conventional CE adaptive control system (see Fig. 1) and adaptive dual control system (see Fig. 2) lies in the parameter estimates transmission. In the case of dual system, both parameter estimates and their accuracy are considered. If the uncertainty of recursively acquired parameter estimates is taken into account, it is possible to calculate the controller output, which ensures the optimal excitation of system for quality identification at keeping the cautious character of controlling signal. This approach can markedly improve the quality of control of systems with small a priori information and high level of uncertainty.

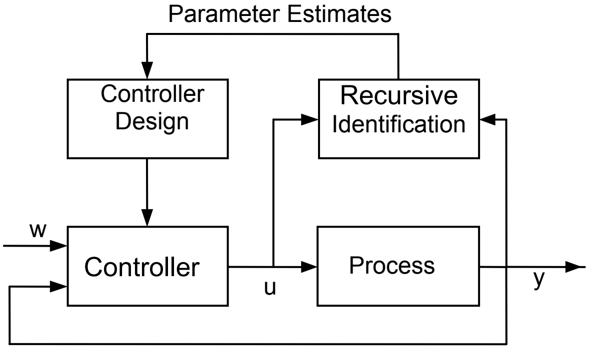

Fig.1 CE adaptive control system

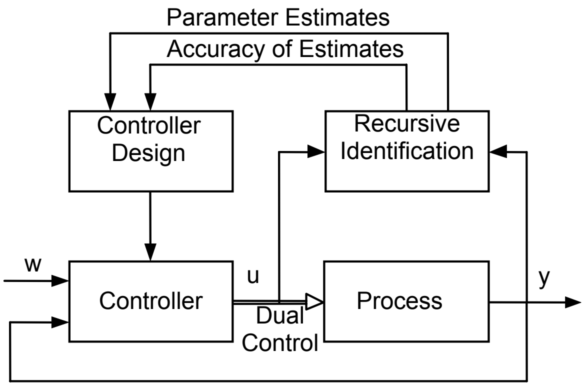

Fig.2 Dual adaptive control system

2. Bicriterial approach

Two criteria minimization method called bicriterial approach is based on sequential minimization of two cost functions for dual control corresponding to two aims of dual control (see Fig. 3). The first function is control losses J_k^c and its optimum after minimization is the *cautious control* action $u_c(k)$. This cautious controller results in a control signal with a magnitude smaller than that which an ordinary CE controller would achieve so there are smaller overshoots after the start of a process. The second cost function J_k^a which stands for parametric uncertainty is minimized around the cautious control value in the Ω_k domain. The resulting control action value is given as a compromise of optimization of two criteria when the magnitude of the excitation is given by the size of domain Ω_k. It is suitable to define

these constraints symmetrically around the cautious control value $u_c(k)$ by the value of parameter θ_k representing magnitude of the additional excitations. Finally, we obtain the dual controller by bicriterial optimization:

$$u(k) = \underset{u(k)\in\Omega_k}{\arg\min}\, J_k^a \tag{1}$$

$$\Omega_k = \left[u_c(k) - \theta(k); u_c(k) + \theta(k)\right] \tag{2}$$

$$\theta(k) = \eta\, \mathrm{tr}\{\boldsymbol{C}(k)\}; \quad \eta \geq 0 \tag{3}$$

$$u_c(k) = \underset{u(k)}{\arg\min}\, J_k^c . \tag{4}$$

The amplitude of excitations is dependent on the value of the selectable parameter η and the trace of covariance matrix $\boldsymbol{C}(k)$.

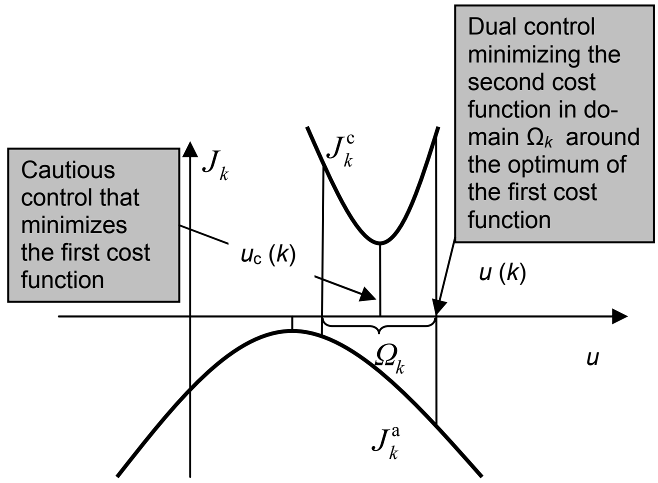

Fig.3 Optimization of two cost functions

3. Dual modification of CE controller

In the case of explicit STC it is possible to use to design of a dual controller independently of the structure of the standard CE adaptive controller. A dual controller obtained by this way can be used together with any CE controller with indirect adaptation (e.g. pole placement, LQG, digital Ziegler-Nichols, predictive, generalized minimum-variance etc.). It is introduced as additional unit modifying the CE control signal to the dual control one. Improvement of the control performance is the result of this simple modification.

Now consider a single input – single output (SISO) system described by the linear stochastic differential equation (discrete time input/output model)

$$\begin{aligned} y(k+1) &= b_1 u(k) + \ldots + b_{nb} u(k - nb + 1) - \\ &\quad - a_1 y(k) - \ldots - a_{na} y(k - na + 1) + \xi(k) = \\ &= b_1 u(k) + \boldsymbol{\Theta}_0^T \boldsymbol{\Phi}_0(k) + \xi(k) = \boldsymbol{\Theta}^T \boldsymbol{\Phi}(k) + \xi(k) \end{aligned} \tag{5}$$

where

$$\boldsymbol{\Theta}^T = \left[b_1, \ldots, b_n, a_1, \ldots, a_n\right] = \left[b_1 \vdots \boldsymbol{\Theta}_0^T\right] \tag{6}$$

is the ARX model parameter vector and

$$\begin{aligned} \boldsymbol{\Phi}^T(k) &= \left[u(k), \ldots, u(k-n+1), -y(k), \ldots, -y(k-n+1)\right] = \\ &= \left[u(k) \vdots \boldsymbol{\Phi}_0^T\right] \end{aligned} \tag{7}$$

is the regression vector ($y(k)$ is the process output variable, $u(k)$ is the controller output variable). The noise sequence $\xi(k)$ has variance σ_ξ^2. A simple *recursive least squares* identification method is used to estimate the plant parameters. The vector of parameter estimates is updated as

$$\hat{\boldsymbol{\Theta}}(k+1) = \hat{\boldsymbol{\Theta}}(k) + \frac{\boldsymbol{C}(k)\boldsymbol{\Phi}(k)}{\boldsymbol{\Phi}^T(k)\boldsymbol{C}(k)\boldsymbol{\Phi}(k) + \sigma_\xi^2}\hat{e}(k+1) \tag{8}$$

where

$$\hat{e}(k+1) = y(k+1) - \hat{\boldsymbol{\Theta}}^T(k)\boldsymbol{\Phi}(k) \tag{9}$$

stands for prediction error. Square covariance matrix is updated in each sampling period according to

$$\boldsymbol{C}(k+1)=\boldsymbol{C}(k)-\frac{\boldsymbol{C}(k)\boldsymbol{\Phi}(k)\boldsymbol{\Phi}^T(k)\boldsymbol{C}(k)}{\boldsymbol{\Phi}^T(k)\boldsymbol{C}(k)\boldsymbol{\Phi}(k)+\sigma_\xi^2} \tag{10}$$

The following notation for covariance matrix is introduced for on-coming manipulations

$$\boldsymbol{C}(k)=E\left\{\left[\boldsymbol{\Theta}-\hat{\boldsymbol{\Theta}}(k)\right]\left[\boldsymbol{\Theta}-\hat{\boldsymbol{\Theta}}(k)\right]^T|\Im_k\right\}=$$

$$=\begin{bmatrix} c_{11}(k) & \dots & c_{1n}(k)\\ \vdots & \ddots & \vdots \\ c_{n1}(k) & \dots & c_{nn}(k)\end{bmatrix}=\begin{bmatrix} c_{b_1}(k) & \boldsymbol{c}_{b_1\boldsymbol{\Theta}_0}^T(k)\\ \boldsymbol{c}_{b_1\boldsymbol{\Theta}_0}(k) & \boldsymbol{C}_{\boldsymbol{\Theta}_0}(k)\end{bmatrix} \tag{11}$$

The set of process outputs and inputs available at time k is denoted as

$$\Im_k=\{y(k),...,y(0),u(k-1),...,u(0)\};$$

$$k=1,...,N-1;\ \Im_0=\{y(0)\}$$

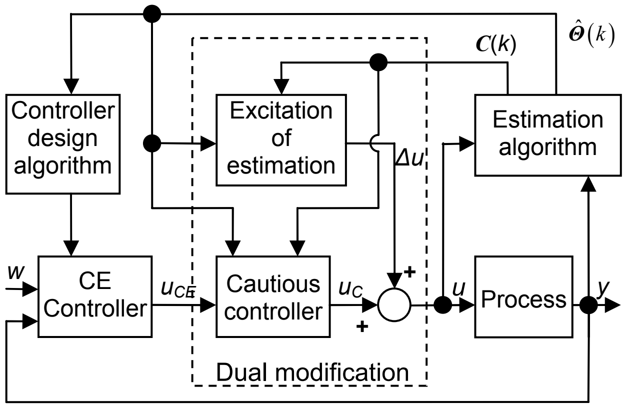

Fig.4 Detailed scheme of an adaptive dual control system (Δu represents the optimal excitation)

Nominal system output for CE controller is

$$\hat{y}(k+1)=\hat{b}_1(k)u_{\mathrm{CE}}(k)+\hat{\boldsymbol{\Theta}}_0^T(k)\boldsymbol{\Phi}_0(k), \tag{12}$$

where $u_{CE}(k)$ is CE controller output signal. Dual control cost functions are given as

$$J_k^{\mathrm{c}}=E\left\{\left[\hat{y}(k+1)-y(k+1)\right]^2|\Im_k\right\} \tag{13}$$

and

$$J_k^{\mathrm{a}}=-E\left\{\left[y(k+1)-\hat{\boldsymbol{\Theta}}^T(k)\boldsymbol{\Phi}(k)\right]^2|\Im_k\right\} \tag{14}$$

Substituting equations (5) and (12) into equation (13) and minimization of modified equation (13) leads to the cautious control law

$$u_{\mathrm{c}}(k)=\frac{\hat{b}_1^2(k)u_{\mathrm{CE}}(k)-\boldsymbol{c}_{b_1\boldsymbol{\Theta}_0}^T(k)\boldsymbol{\Phi}_0(k)}{\hat{b}_1^2(k)+c_{b_1}(k)} \tag{15}$$

Minimization of equation (1), with constraints according to equation (2), leads to

$$u(k)=u_{\mathrm{c}}(k)+\theta(k)\operatorname{sgn}\left\{J_k^{\mathrm{a}}\left[u_{\mathrm{c}}(k)-\theta(k)\right]-J_k^{\mathrm{a}}\left[u_{\mathrm{c}}(k)+\theta(k)\right]\right\} \tag{16}$$

and finally after next modification resulting dual control law is in the form [6]

$$u(k)=u_{\mathrm{c}}(k)+\theta(k)\operatorname{sgn}\left\{c_{b_1}(k)u_{\mathrm{c}}(k)+c_{b_1\boldsymbol{\Theta}_0}^T(k)\boldsymbol{\Phi}_0(k)\right\} \tag{17}$$

The detailed structure of an adaptive control system with a dual control unit is shown in Fig. 4.

4. Interconnected tanks

Above-mentioned dual approach was verified by control of liquid level in the interconnected cylindrical tanks.

4.1 Mathematical model

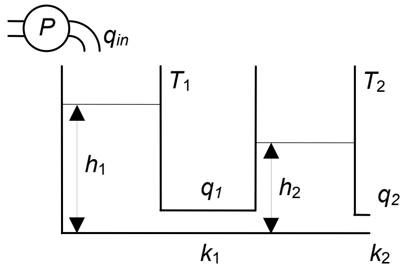

Fig.5 Scheme of two interconnected tanks

A scheme of two interconnected tanks is presented in the Fig. 5. The system consists of two interconnected cylindrical tanks T_1 and T_2 and a pump P which is responsible for inflow to the tank T_1. The liquid level heights in the tanks T_1 and T_2 are h_1 and h_2 respectively. The inflow produced by the pump is q_{in}, flow between tanks is q_1 and the outflow is q_2. The pipe between tanks and the outflow pipe are described by constants k_1 and k_2 respectively. The model can be described by the following system of nonlinear partial differential equations

$$q_{in}=q_1+\frac{dV_1}{dt};\quad q_1=q_2+\frac{dV_2}{dt}$$
$$q_1=k_1\sqrt{|h_1-h_2|}\operatorname{sgn}(h_1-h_2);\quad q_2=k_2\sqrt{h_2} \tag{18}$$

where V_1 and V_2 are capacities of liquid in the tanks T_1 and T_2.

The system can be considered as a single input single output system (SISO) where the input is inflow q_{in} and output is liquid level h_2. This configuration was used in the experiments described in the following sections.

4.2 Real-time laboratory model DTS200

Control experiments were performed using real-time laboratory model Amira DTS200 – Three Tank System. The scheme of this model is shown in Fig. 6.

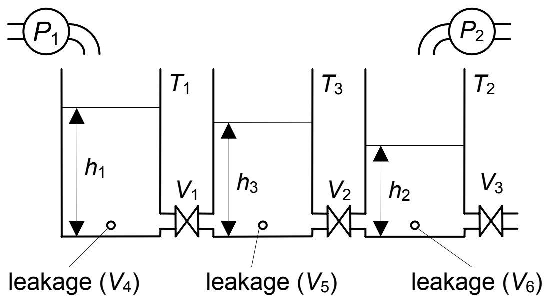

Fig.6 Scheme of Amira DTS200

The system consists of three interconnected cylindrical tanks, two pumps, six valves, pipes, measurement of liquid levels and other elements. Valves V_2 and V_4 were fully closed during the experiments, valve V_1 was fully opened and valve V_5 was partially opened. The valve positions did not change during the experiments. This configuration leads to the same model as described in the previous section. The controlled signal (y) was the height of the liquid level in the middle tank ($y = h_3$). This level was controlled by the control voltage of the pump P_1 (u).

Due to the characteristics of the valves, pipes and pump, the system behaviour contains more nonlinearities than the mathematical model described by equations (18). This can be seen from the static characteristics shown in Fig. 7.

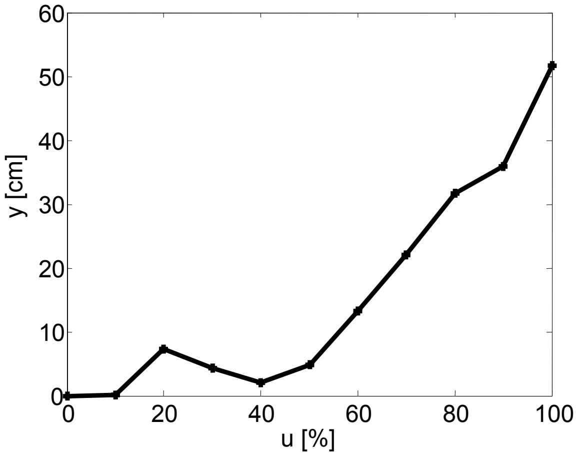

Fig.7 Static characteristic of the controlled system

4.3 Control of laboratory model

Several different adaptive control algorithms were used to control the described system and results of two of them are presented later in this chapter. Both controllers are based on recursive least squares on-line identification combined with the pole placement control law. Controlled system was modelled as a discrete second order linear system. A sampling period of T_0 = 10 s was used for all experiments and initial parameter estimates were set without using a priori information about controlled system. Dual controllers' performances were compared to the performance of the same controllers without dual modification (CE).

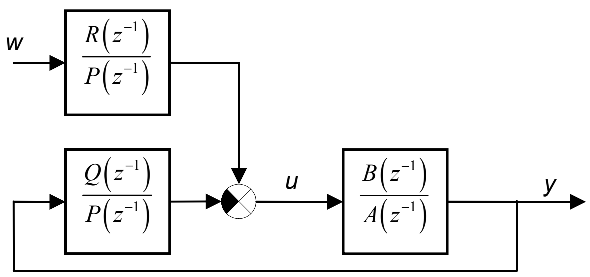

Fig.8 Closed loop 2DOF control system

The first controller uses two degree of freedom (2DOF) structure (see Fig. 8), where

$$A\left(z^{-1}\right)=1+a_1z^{-1}+a_2z^{-2};\quad B\left(z^{-1}\right)=b_1z^{-1}+b_2z^{-2} \tag{19}$$

are polynomials of the controlled process model and

$$Q\left(z^{-1}\right)=q_0+q_1z^{-1}+q_2z^{-2}$$
$$P\left(z^{-1}\right)=\left(1-z^{-1}\right)\left(1+p_1z^{-1}\right);\quad R=r_0=\frac{1}{b_1+b_2} \tag{20}$$

are polynomials of the controller. Controller parameters are then computed by solving of the polynomial equation

$$A\left(z^{-1}\right)P\left(z^{-1}\right)+B\left(z^{-1}\right)Q\left(z^{-1}\right)=D\left(z^{-1}\right) \tag{21}$$

where the desired characteristic polynomial of the closed loop was set according to equation

$$D\left(z^{-1}\right)=1+d_1z^{-1}+d_2z^{-2} \tag{22}$$

The coefficients of polynomial (22) were chosen as $d_1=-1.6, d_2=0.64$.

A solution of the polynomial equation (21) by the uncertain coefficients method leads to a system of linear equations

$$\begin{bmatrix} b_1 & 0 & 0 & 1 \\ b_2 & b_1 & 0 & a_1-1 \\ 0 & b_2 & b_1 & a_2-a_1 \\ 0 & 0 & b_2 & -a_2 \end{bmatrix}\begin{bmatrix} q_0 \\ q_1 \\ q_2 \\ p_1 \end{bmatrix}=\begin{bmatrix} 1-a_1 \\ a_1-a_2 \\ a_2 \\ 0 \end{bmatrix} \tag{23}$$

and their solution gives controller parameters. The CE control law is given by the equation

$$\begin{aligned} u_{CE}(k) &= r_0w(k)-q_0y(k)-q_1y(k-1)-q_2y(k-2) \\ &\quad -(1-p_1)u(k-1)+p_1u(k-2) \end{aligned} \tag{24}$$

The selectable parameter in equation (3) was chosen as η=30. The CE version of this controller is further referenced as *pp1* and its dual version as *pp1_d*. The control performances of *pp1* and *pp1_d* controllers can be seen in Fig. 9 and Fig 10.

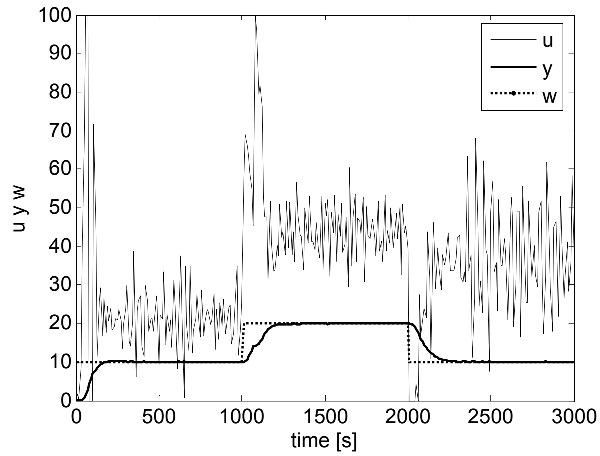

Fig.9 Control performance of *pp1* controller

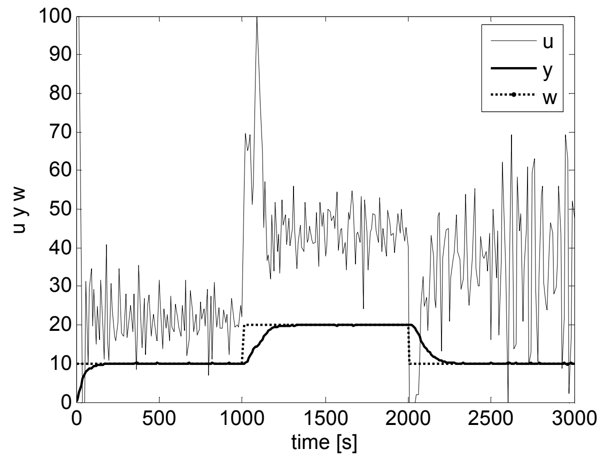

Fig.10 Control performance of dual *pp1_d* controller

The second presented self-tuning controller is based on pole placement controller *pp2b1* from the Self-Tuning Controllers Simulink Library [7], [8]. The idea of this controller is to make the dynamic behaviour of the closed loop similar to that of the second order continuous system with characteristic polynomial as stated by equation

$$D(s) = s^2 + 2\xi\omega_n s + \omega_n^2 \qquad (25)$$

If the polynomial $D(z^{-1})$ is chosen in the form

$$D(z^{-1}) = 1 + d_1 z^{-1} + d_2 z^{-2} \qquad (26)$$

then the following relations to calculate the coefficients for a sampling period T_0 can be derived

$$d_1 = -2\exp(-\xi\omega_n T_0)\cos(\omega_n T_0\sqrt{1-\xi^2})\ ; \quad \text{for } \xi \le 0$$

$$d_1 = -2\exp(-\xi\omega_n T_0)\cosh(\omega_n T_0\sqrt{1-\xi^2})\ ; \quad \text{for } \xi \rangle 0 \qquad (27)$$

$$d_2 = \exp(-2\xi\omega_n T_0)$$

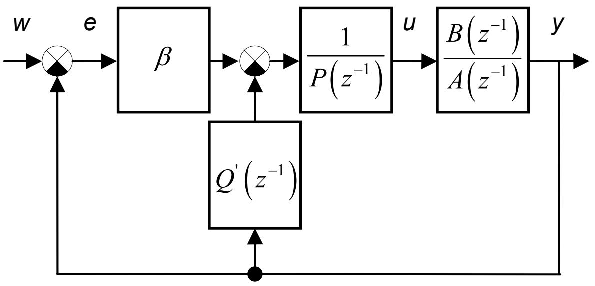

Fig.11 Closed loop control system with *pp2* controller

Structure of control circuit with this controller is shown in Fig. 11. From this figure is obvious that characteristic polynomial has the form

$$A(z^{-1})P(z^{-1}) + B(z^{-1})\left[Q^{'}(z^{-1}) + \beta\right] = D(z^{-1}) \qquad (28)$$

and while the polynomial $P(z^{-1})$ has the same form as polynomial (20) for controller (24), the second polynomial $Q'(z^{-1})$ takes the form

$$Q^{'}(z^{-1}) = (1 - z^{-1})(q_0^{'} - q_2^{'} z^{-1}) \qquad (29)$$

A solution of the polynomial equation (28) by the uncertain coefficients method leads to a system of linear equations

$$\begin{bmatrix} b_1 & 0 & b_1 & 1 \\ b_2 - b_1 & -b_1 & b_2 & a_1 - 1 \\ b_2 & b_2 - b_1 & 0 & a_1 - a_2 \\ 0 & b_2 & 0 & -a_2 \end{bmatrix} \begin{bmatrix} q_0 \\ q_2 \\ \beta \\ p_1 \end{bmatrix} = \begin{bmatrix} d_1 + 1 - a_1 \\ d_2 + a_1 - a_2 \\ -a_2 \\ 0 \end{bmatrix} \qquad (30)$$

and their solution gives controller parameters. The CE control law is then given by equation

$$u_{CE}(k) = -(q_0' + \beta)y(k) + (q_0' + q_2')y(k-1) - q_2' y(k-2) - \\ -(p_1 - 1)u(k-1) + p_1 u(k-2) + \beta w(k) \qquad (31)$$

Controller parameters were set to ξ=1 and ω_n=0.05, which leads to asymptotic step responses. The CE version of this controller is further referred to as *pp2* and its dual modification as *pp2_d*. The control performance of *pp2* and *pp2_d* controllers can be seen in Fig. 12 and Fig. 13.

Nonlinearities and changes of the behaviour of controlled system can be observed from the figures presented in this chapter. Inspecting Fig 9, it can be seen that reference signal of 10 cm caused to the control signal of about 20% in the first part but to 40% in the last part. Thus, the gain of the system has decreased to the half during control course.

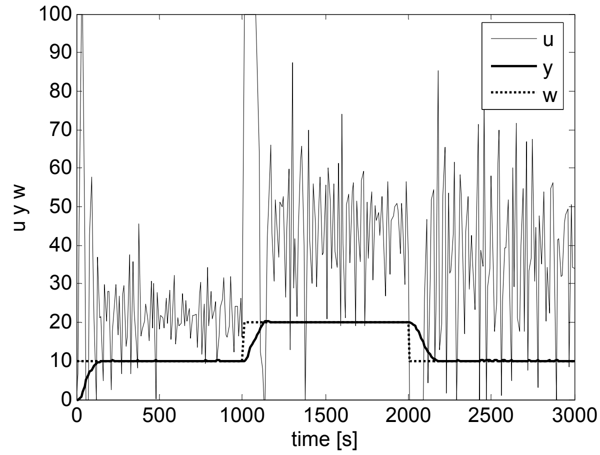

Fig.12 Control performance of *pp2* controller

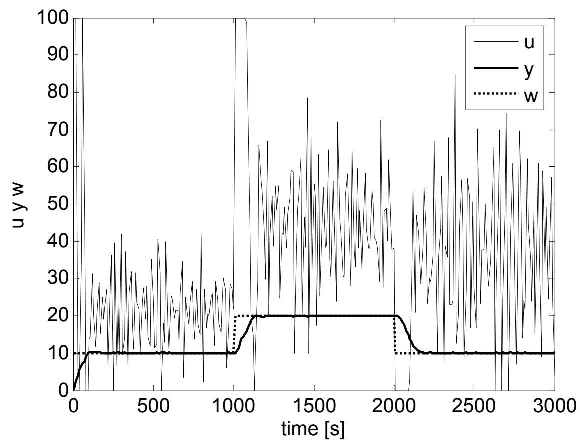

Fig.13 Control performance of dual *pp2_d* conroller

4.4 Comparison of control performance using summing criteria

The performances of individual controllers were compared not only by investigating graphs of performance of controller and process output signal, but also by mathematical criteria. Four criteria were used to compare control courses obtained by individual controllers

$$S_{e2} = \frac{1}{N}\sum_{k=1}^{N}\left[w(k) - y(k)\right]^2; \quad S_{ea} = \frac{1}{N}\sum_{k=1}^{N}\left|w(k) - y(k)\right| \\ S_{u2} = \frac{1}{N-1}\sum_{k=1}^{N-1}\left[\Delta u(k)\right]^2; \quad S_{ua} = \frac{1}{N-1}\sum_{k=1}^{N-1}\left|\Delta u(k)\right| \qquad (32)$$

Values of individual criteria for *pp1* and *pp1_d controllers* are compared in table 1 and for *pp2* and *pp2_d* controllers in table 2.

Criteria S_{e2} and S_{ea} are based on control error. Sum of squares of control error and sum of absolute values of control error were used to obtain S_{e2} and S_{ea} respectively. These criteria represent accuracy of control process. Criteria S_{u2} and S_{ua} are based on changes control signal. Sum of squares of control sequence and sum of absolute values of control sequence were used to obtain S_{u2} and S_{ua} respectively. These criteria represent demands for actuators. Value N was selected to cover whole control process (N=301).

Usage of *pp2* controllers led to better accuracy of control process but demands on the actuator were higher. Usage of dual modification led to better performance of controller in almost all tested cases.

controller	S_{e2}	S_{ea}	S_{u2}	S_{ua}
pp1	7,12	1,03	261	11,4
pp1_d	5,49	0,86	225	10,7
Improvement	23%	17%	14%	6%

Tab.1 Values of criteria for the control courses of *pp1* and *pp1_d* controllers

controller	S_{e2}	S_{ea}	S_{u2}	S_{ua}
pp2	5,70	0,81	599	17,6
pp2_d	4,71	0,70	581	17,7
Improvement	17%	14%	3%	-1%

Tab.2 Values of criteria for the control courses of *pp2* and *pp2_d* controllers

Conclusions

Dual control using bicriterial approach was verified and compared with some other standard adaptive control approaches in real-time conditions by controlling a laboratory model. Examples of control of nonlinear and time varying DTS200 Tank System were shown. Despite the fact that the nonlinear system was modelled by a linear model, real-time experiments demonstrated that the dual controller can be suitable for control of nonlinear systems.

Acknowledgements

This work was supported in part by the Ministry of Education of the Czech Republic under grant MSM 7088352101, and in part by the Grant Agency of the Czech Republic under grants No. 102/05/0271 and No. 102/06/P286.

References

[1] FELDBAUM, A. A.: Dual control theory. Automation and Remote Control, **21**, 1960, 874-1039.

[2] FELDBAUM, A. A.: Optimal Control Systems. New York, Academic Press, New York, USA, 1965.

[3] WITTENMARK, B.: Adaptive dual control methods: an overview. In: Proc. 5th IFAC Symposium on Adaptive Systems in Control and Signal Processing, Budapest, Hungary, 1995, 62-75.

[4] FILATOV, N. M., UNBEHAUEN, H.: Survey of adaptive dual control methods. IEE Proc. Control Theory and Applications, **147**, 2000, 118-122.

[5] WITTENMARK, B.: Adaptive dual control. UNESCO Encyclopedia of Life Support Systems, 6.43.15.6 EOLSS Publishers, Oxford, United Kingdom, http: //www.eolss.net/, 2003.

[6] FILATOV, N. M., UNBEHAUEN, H.: Adaptive Dual Control. Theory and Application. Berlin Heidelberg New York, Springer-Verlag, 2004

[7] BOBÁL, V., CHALUPA, P.: Self-tuning Controllers Simulink Library. http: //www.utb.cz/stctool/, 2003.

[8] BOBÁL, V., BÖHM, J., FESSL, J., MACHÁČEK, J.: Digital Self-tuning Controllers: Algorithms, Implementation and Applications. London, Springer-Verlag, 2005.

prof. Ing. Vladimír Bobál, CSc.
Ing. Petr Chalupa, Ph.D.
prof. Ing. Petr Dostál, CSc.

Department of Process Control
Faculty of Applied Informatics
Tomas Bata University in Zlín
Nad Stráněmi 4511, 760 05 Zlín
Czech Republic
Tel.: +420 57603 5197
Fax: +420 57603 5279
e-mail: bobal@fai.utb.cz
chalupa@fai.utb.cz
dostalp@fai.utb.cz